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SMALL MANUFACTURING BUSINESSES

A PRELIMINARY REPORT ON A PILOT SURVEY

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I

Objectives and Methods

The Purpose of the Survey

Information about small and medium businesses, which form such a large proportion of the total number of firms in this country, is not entirely lacking. The Ministry of Labour regularly gives figures of employment by size groups of firms, and the reports of the Commissioners of Inland Revenue provide figures of profits and profit margins for firms of different size. The Censuses of Production and Distribution at intervals provide data about the value of output and a breakdown of inputs. The sets of data gathered by Government Departments do not, however, cover all or even the most important aspects of their activities, and cannot be combined into a coherent picture of the working of the small business unit. If this socially and economically important sector of the business community is to be investigated more thoroughly, it can only be done by gathering the relevant facts from the men who are managing these businesses. To this end a positive contribution could be made by case studies of small firms or industries in which small firms predominate, but these have rarely been attempted. For an alternative we can turn to the survey method, which has yielded interesting results in other areas of economic and social research. The survey, in principle at least, would seem to be able to provide information on a broader scale than case studies, though with some loss of individual detail. At the same time survey data are more suited to economic analysis than the data collected by government departments. For they can simultaneously describe the performance, behaviour and attitudes of each unit of the sample, thus providing a link between the recorded transactions of the business and the businessman's outlook and attitudes.

The potential value of such a survey of business may not be seriously disputed, but there will be doubts whether businessmen can be persuaded to co-operate and give not only the accounts of their business but also much other information which they would not normally reveal to outsiders. We shared these doubts but experience with the annual surveys of income and savings, which included the financial accounts of several hundreds of self-employed persons, had taught us that a business survey need not be regarded as a hopeless undertaking. Since it would be a pioneering and hence a very risky venture, and since surveys are in any case expensive tools of research, it was clearly advisable to experiment on a small scale; small enough to keep costs low and yet sufficient to serve as a test of the practicability of a bigger enquiry.[1]

The design of the question schedule was a crucial stage in the preparatory work which determined and limited the objective of the survey. Almost inevitably it represented a compromise between conflicting aims : of probing more deeply into particular aspects of business behaviour, which might well reduce the willingness of businessmen to co-operate, and of collecting wide enough information about the economic environment in which small businesses operate, at the risk of introducing some vagueness into the answers. The actual compromise adopted is briefly set out in the following paragraphs.

A summary of the activities of a business is usually given in its annual accounts. The balance sheet and the trading and profit and loss account give essential information about its financial structure and economic performance, and are naturally the hard core of the survey. But it is not sufficient to collect sets of accounting data. Without some knowledge of the nature of the business, its products and markets, and its recent history, the financial data cannot adequately be interpreted, and the survey method seems well suited to provide such essential background information.

Our first objective, then, was to collect the firms' accounts. Balance sheets present the framework for measuring size and composition of assets, net worth and various liquidity and financial gearing ratios. The sum total of assets serves (together with employment and turnover) as a useful measure of size, and is clearly relevant also for calculating profitability and efficiency in resource utilisation. Size may then be related to economic performance and the composition of assets and liabilities, a relationship about which economists have put forward various views. Is it true that small firms tend to be more illiquid, to have a smaller proportion of owned capital to borrowed capital, and to have a lower ratio of fixed assets to total assets than large firms, or that they employ less capital per man or have larger profit margins ? Balance sheet data are easily available for all public companies—usually the larger type of company—but not for sole owners, partnerships and private companies. If the survey method would yield an adequate number of balance sheets for the private companies and smaller concerns, this by itself would be sufficient justification for it, since it would enable a comparison to be made between the capital structure of this important sector of the economy and that of public companies.

The trading and profit and loss accounts reveal the cost structure of the firms concerned and their profitability. If profitability can be taken as a symptom of efficiency, or if other measures of efficiency can be derived from the accounts, it may be possible to see how far efficiency is related to size. There should in fact be no need to elaborate further the importance of having a sample of accounts of what we loosely call small businesses.

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[1] The preparation of this article has been made possible by a grant under the Conditional Aid Scheme for the use of Counterpart funds derived from United States Economic Aid.

The second objective of the survey is to fill in the background against which the accounts must be seen, or in other words to get a picture of the economic environment in which the sample firms are working. This background information may be grouped under three main headings.

(*a*) *Market structure*. Any attempt to go thoroughly into the question of the market structure of our firms would have required not only a whole question schedule to itself, but also detailed records of sales and prices which we could not hope to collect. We had to be satisfied with a minimum of information about the main markets and customers, about number and size of rivals, the business men's views about competitive pressure and how they met it, about membership of trade associations and their attitudes toward them. All this cannot easily be quantified or incorporated into a single formula of market structure, but we hope to show that it is of interest as such and for the interpretation of the financial data.

(*b*) *Ownership structure*. The legal form of a business and the nature of its controlling interest probably have some bearing on its market behaviour, on the form of the accounts and the way in which profits are appropriated. Similarly the degree to which external finance is used or sought may be influenced by legal form or ownership structure. Family businesses are said to be more anxious to avoid forms of external finance which would potentially or actually reduce the family's control, while private companies with more diffuse ownership have less hesitation on this score. Moreover a subsidiary or a firm associated with other firms is likely to differ in market or financial behaviour from a fully independent firm. Information about legal status and ownership is therefore essential for an understanding of the firms' accounts and of the attitudes of the business man.

(*c*) *Factors influencing growth*. It seemed unwise to try to collect in a single survey accounts for more than two fiscal years. But clearly this is too short a period to tell us much about the development of the firm. Had it expanded in terms of employment, sales or capacity fairly rapidly to its present state, or merely maintained its position ? Was it hampered by factors on the demand or supply side, or by lack of finance ? Did the Macmillan Gap still exist in the post-war years as a hindrance to the smaller enterprising firm ? Questions of this kind are relevant to an interpretation of the accounts and the role that small and medium businesses play in industry. But here again the number of questions that could be asked to satisfy the demand for knowledge of the economist is very large. Within the bounds of a moderately short interview only a small section of the question-schedule could be devoted to problems of growth. For the post-war years we have attempted to get quantitative information about changes in employment, total assets, capacity and so forth. In addition we have the birth dates of our firms, and although we do not know what happened between their establishment and the early post-war years, it may be possible to draw some conclusions about long-term growth from the distribution of firms by age and size. Bearing in mind conflicting statements about the effect of the economic climate of the post-war years on industry, we thought it pertinent to ask not only about past developments but also about future plans and desired expansion. One set of questions was therefore concerned with the desired size which the individual businessman had in mind for his kind of business, with his plans for expansion and with the degree to which finance had been or was likely to be a limiting factor, and in particular with whether access to external finance was adequate for his plans.

We thus chose three areas in which to seek general information about the environment in which firms work. These areas seemed essential for an interpretation of their economic affairs, but we may have taken the risk of spreading our net too widely. Nevertheless, the interviewing method sets limits to the intensity with which any particular set of questions can be followed up without interfering too much with the central part of the enquiry, namely the collection and analysis of detailed accounts of small business.

A pilot survey cannot by its very nature provide an adequate basis for economic analysis, but it should help to clarify some of the doubts about the applicability of survey techniques to business research and enable us to draw conclusions about the usefulness and adequacy of the various parts that make up its design. In particular it should (*a*) show how far the questions we put to the business men could be answered by them and how far the answers are useful in a descriptive and analytical sense; (*b*) give some notion of interviewer quality and interviewing time required; and (*c*) above all, provide some clues about the degree of co-operation one might expect in the business community.

2. SAMPLING

For financial and administrative reasons we limited the scope of the pilot survey both with regard to regional and to industrial coverage. Birmingham was chosen as the experimental area because it was conveniently near to Oxford and contained many small manufacturing concerns. Distribution and service trades were excluded in order to avoid undue complication of the question schedule and to secure some homogeneity in the small sample. The size range of firms to be included in the sample could not, of course, be defined in advance by any of the more useful economic criteria, and was fixed in terms of employment to cover firms with from 10 to 250 employees. The lower limit was set because we believed—probably wrongly—that accounting information of the very small firms might be inadequate or unobtainable. Manufacturing was defined in the same way as in the Standard Industrial Classification (S.I.C.) except that we excluded certain groups. S.I.C.

Orders I and II, Agriculture, Forestry and Fishing, and Mining and Quarrying, were excluded, and the remaining manufacturing Orders III to XVI were included, with the exception of certain subgroups, namely, Motor Repairers and Garages, Tailoring, Dressmaking, Boot and Shoe Repairers, and Bread and Flour Confectionery. These trades were excluded mainly on the practical grounds that the bulk of the establishments practising them had under 10 employees. It was also decided to exclude those public companies whose shares are quoted on the Stock Exchange, because most of them would be outside our size limits, and in any case the financial structure of quoted public companies is at present the subject of a study by the National Institute for Economic and Social Research. Our population was therefore defined as all other firms, including unquoted public companies.

Our next problem was to find a sampling frame and to devise the sampling procedure. Government Departments such as the Ministry of Labour, the Board of Trade, or the Board of Inland Revenue have, of course, adequate information for this purpose about businesses operating in this country, but unfortunately their various registers were not available to us. On the other hand there seems to be no comprehensive list of firms outside government offices on which one could draw. There is a national list with the Registrar of Companies in Bush House, in which public and private companies are mixed alphabetically, but without consulting the file of every company in the country, it is not possible to get a regional or industrial breakdown. Besides, there is no comparable list of partnerships and sole owners, who also formed part of our population. We were driven, therefore, to draw our population from the classified trades section of Kelly's Birmingham Directory 1954 (K.D.)[1] which in our size range is probably quite comprehensive and, in principle at least, adequate as a sampling frame. Although in most cases it was fairly easy to reconcile the Directory's trade lists with the S.I.C., we were doubtful about a few sections and these may have been included or excluded wrongly. One further difficulty is that many firms appear in more than one trade list, either because they do in fact practise several trades or because their trade goes under several different names. Moreover, classifications under which a firm is listed do not necessarily cover all the trades carried on by the firm. Thus, among the firms excluded as non-manufacturing, there may in fact have been some manufacturing concerns. In addition, as some firms were in more trade lists than others (for whatever reason), they would have a greater chance of selection in our sample. This last difficulty could have been rectified to some extent, but, as we did not intend to aggregate our data to make regional or national comparisons, we did not think it worth doing so. The sample is, we believe, sufficiently ' representative ' (in the sense of covering a wide range of industries and a fair range of size differences) to indicate certain general characteristics of small manufacturing business in Birmingham.

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[1] We should like to acknowledge here the considerable help which we received from the publishers of the Directory.

Assuming that the response rate would be around 50 per cent and that we wanted 100 respondents we required a sample of 200 firms. But K.D. included quoted public companies and firms outside our size range. From the Census of Production, it appeared that about half of all firms in the country, fell in our size range, and so we had to increase the sample from 200 to 400 firms to allow for exclusions by size. It was decided to add another 50 firms to the sample, to allow for the exclusion of quoted public companies not excluded by size, and to leave a margin for any other eventualities. The initial sample, then, consisted of 450 firms, and in addition we drew a sub-sample comprising all the firms in a fairly homogeneous industrial sub-section from which we hoped to get some significant intra-industry comparisons.

The population was split into 14 strata—each stratum consisting of the firms on our K.D. lists which belonged to a particular S.I.C. Order. The number of firms in the sample to be drawn from each stratum depended on the ratio which the total employment of firms employing between 10–250 in each stratum bore to the total employment in all strata of firms in this size range. The simplest procedure would have been to construct the sample by selecting the same proportion of firms from each stratum. This, however, would make no allowance for the fact that firms in different strata may typically be in very different size ranges. The method adopted, of using employment weights, means that the proportionate representation of each stratum varies with the mean size of firm in the stratum. It was chosen, however, because it was felt that the relative importance of an industry was better measured by the employment it gave, than by the number of firms it contained irrespective of their size. Also, as the density of our population was greater at the lower end of our size range, it was desirable that we should sample the upper end relatively more intensively if a good spread through the size groups was to be obtained, and also because a greater variability was to be expected among larger firms. Using the employment weights, a random sample was drawn from each of our 14 strata with the initial sample of 450 firms distributed as shown in Table I.

Table II shows how the initial sample of 450 firms was reduced to 142 firms. The special sample of an industrial sub-group yielded, for various reasons, only six firms to be interviewed and only three completed schedules and was, therefore, merged with the main sample.

In the main sample it was estimated that 200 of the sample firms would be outside our size range. The actual exclusions in the main sample numbered 213. But the margin of 50 firms for other eventualities was not nearly enough, mainly because the proportion of quoted public companies in our initial sample was much higher than the national figures would have led one to expect. This may have been due to the fact that they would be more likely than other firms to appear under more than one trade classification in K.D., and would hence have a higher chance of selection. Thus our effective sample consisted only of 148 firms instead of the 200 that we had expected.

TABLE I

Initial Sample

	Standard Industrial Classification Order	*No. of firms in K.D.*	*No. of firms in sample*	*No. of firms as a % of total firms in sample*
III	Treatment of Non-Metalliferous Mining Products other than coal	372	10	2.2
IV	Chemicals and Allied Trades	399	18	4.0
V	Metal Manufacture	578	30	6.7
VI	Engineering, Shipbuilding and Electrical Goods	2,324	54	12.0
VII	Vehicles	454	54	12.0
VIII	Metal Goods not elsewhere specified	2,793	135	30.0
IX	Precision Instruments, Jewellery, etc. ...	753	10	2.2
X	Textiles	178	7	1.6
XI	Leather, Leather Goods and Fur	103	5	1.1
XII	Clothing	87	3	0.7
XIII	Food, Drink and Tobacco	178	43	9.6
XIV	Manufactures of Wood and Cork	749	19	4.2
XV	Paper and Printing	702	21	4.7
XVI	Other Manufacturing Industries...	464	41	9.1
	Total	10,134	450	100

TABLE II

Construction of Effective Sample from Initial Sample

	Initial Sample	*Not in size range 10–250 employees*	*Quoted public companies*	*Located outside Birmingham*	*Non-manufacturing*	*Total number excluded*	*Effective Sample*
Main sample	450	213	74	4	17	308	142
Sub-group sample	54	19	27	1	1	48	6

3. INTERVIEWING

Field work began early in January, 1955. The weather was cold with several inches of snow on the ground, many firms were stocktaking after the Christmas sales, not a few of the Managing Directors were away ill or still on holiday, and a fair proportion of the firms' accounts were not available. Obviously mid-winter is not the ideal interviewing period.

We assumed that a survey of business involving the collection of accounting data needed interviewers who were highly qualified especially since they might have to build up the accounts by a long series of detailed questions. (Less knowledge of accountancy and business concepts would be required where the financial data could be copied or posted to the sponsoring institution.) Accordingly our interviewers were carefully chosen,[1] thoroughly briefed, and asked to do a few trial interviews before they tackled the sample firms. We were available during the main interviewing period to give advice and help, and we set up temporary headquarters locally in Birmingham University. To gain first hand acquaintance with the snags of interviewing we carried out a fair number of interviews ourselves. A few days before the interviewers were due to go into the field we wrote to each firm explaining the purpose of the inquiry, saying that an interviewer would be calling shortly, and asking for co-operation.

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[1] We were fortunate to obtain the services of five experienced interviewers from the Social Survey and of five third year students of Birmingham University. We very much appreciate the assistance given to us by the Social Survey and by Professor Sargent Florence and his colleagues of Birmingham University, and wish to thank all the interviewers for carrying out their work under very trying conditions.

One factor for which the trial survey did not prepare us was the time it often takes to arrange an effective interview. Sometimes the Managing Director was out, at other times he was too busy; some firms said they would remain too busy to be interviewed for several weeks ahead. Many firms did not have their accounts readily available, stating that these were always left with the accountants. Where this was the case difficulties were experienced, on occasion, in procuring copies. Thus, even from some firms which in principle seemed willing to co-operate it took much time and effort to get either an effective interview or the accounts, and we were still visiting a few firms after Easter. The extension of the interview period and the need for follow-up in some difficult cases should be borne in mind in evaluating response rates, and in estimating the interviewing effort and expense of a larger survey.

4. EDITING

Once we had collected the information, each questionnaire had to be edited for internal consistency. Then certain broad questions had to be settled, such as how to treat tax and profits so as to render the figures we produced for different firms comparable. We had earlier decided to transfer our information to hand-punched Copeland-Chatterson cards. With the small number in our sample machine punching would have been costly; even the value of hand punching is somewhat doubtful in cases of small numbers and we might have found suitably prepared work sheets quicker. As it was we only punched on to cards that part of our questionnaire which dealt with information not drawn from accounts, most of which we had already pre-coded. In order to make accounts more readily comparable we reduced them to a standard form on work sheets. There is an astonishing variety of ways in which accountants present their figures. Much of this, of course, is due to the fact that different types of accounting information are useful to different types of firms. Even so, a greater standardisation of accounting practices would seem to have advantages. Certainly, in transferring the accounts[1] to a standard form we had certain difficult decisions to make about the disposition of particular items.

We can illustrate the kind of difficulties by reference to some of the concepts we shall actually adopt, in particular to profit concepts, which were especially troublesome. After some consideration we decided to employ three main profit concepts. The first we call ' gross profit ' and define as profits before tax and dividends or drawings.[1] The major difficulty with this concept lies in the fact that, for the sake of certain tax advantages, in many private limited companies which are director controlled the directors vote themselves, within certain limits, as large a remuneration as possible. Yet the size of the balance available for gross profit depends, *inter alia*, on how much has been deducted for directors' remuneration. One might reasonably argue, accordingly, that often in the case of director-controlled companies directors' remuneration exceeds genuine wage cost and some of it ought to be counted as gross profit. The difficulty is to say how much in any particular case. Ideally, we should like to add back directors' remuneration to gross profit in all cases, and then deduct a hypothetically correct directors' wage. But obviously we have no data on which to base an estimate of this correct wage. For most purposes, therefore, we shall use a simple gross profit concept, i.e. we shall tacitly assume that all directors' remuneration is a wage cost : on the whole this would seem to be better than the other practicable alternative of adding back the remuneration on the assumption that it was all profit. We shall, however, for some purposes, adopt this alternative as well.

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[1] We are greatly indebted to Mr. J. E. Critchley for his considerable help with the many accounting problems which arose in the planning and analysis of the survey.

The second of our three concepts we call ' net profit ' and define as gross profit less tax. This concept is also subject to the difficulty we have just described, so that for certain comparisons we shall also add back directors' remuneration. But there is the further difficulty of what tax to deduct. Firms in any one year may deduct from their gross profit an estimate of the tax due (to be paid the following year), tax previously underestimated and now due to be paid, and perhaps an additional amount to be added to their tax reserve, in case they have under-estimated their future liabilities. Hence the amount of tax actually paid by the firm (which can also reflect past losses offset against present profits) may bear no relation at all to its current gross profit. To arrive at net profit therefore we shall deduct an imputed tax based on the current year's gross profit.

The third of our concepts of profit we call ' retained profit '. In obtaining a figure for retained profits a problem arises, as before, in connection with directors' remuneration. Directors in private limited companies, especially those which are director controlled, often do not draw the entire remuneration with which they are credited. Where this occurs the undrawn balance appears in the accounts as a ' loan ' to the firm. Just as there sometimes might be a theoretical case for treating part of directors' remuneration as gross profit, so there might be a case for treating all or part of a director's loan to his firm as retained profit. Nevertheless, for the same reasons as led us for some purposes to treat all directors' remuneration as wage cost, we must clearly treat directors' loans as distinct from retained profit. Consequently we define ' retained profit ' simply as net profit minus dividends or drawings. Net profit, as described above, is arrived at by deducting an *imputed* tax liability and so, consequently, is retained profit. But for some purposes we need to consider just what the firm did in fact retain in the business. Where this is so we shall deduct actual tax liabilities instead of an imputed tax figure, to arrive at what we shall call ' actual retained profit '.

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[1] Drawings of sole owners or partners.

We encountered similar difficulties in evolving suitable measures of ' the rate of profit '. In the end we decided to express our various profit measures as ratios on several capital and turnover categories. In general, we shall use 'total assets'[1] as the best measure of ' capital employed ', although we shall also express profits as a ratio on share capital and on net worth (equities and reserves). None of our firms had a substantial holding of marketable securities, otherwise, there might have been some difficulty in using ' total assets ' as a measure of ' capital employed '. We prefer assets in this connection to net worth, because assets reflect also the contribution made by loan finance, etc., while net worth only includes, so to speak, the owner's ' stake in the business '.

Even, however, in connection with assets, the problem of deriving comparable figures for different firms arises. For instance, about one-third of our sample rented their land and buildings, while the remainder owned them. The proportion renting their premises may be higher in Birmingham than elsewhere, since there has recently been a considerable amount of ' compulsory purchase ' by the municipality to further the new town plan. We have not sufficient data to impute a rent to the firms which own their premises, and generally we shall use the assets figures as they are. But as there is no way of ensuring general comparability, we shall present some figures for assets less land and buildings, and also on some occasions we shall omit the firms which rent their premises. There were other difficulties in making comparisons between the firms, but most of these will be commented on later in the text as they arise.

5. RESPONSE RATES

The sample population consisted, as already mentioned, of 148 firms. Of these, 68 firms (46 per cent) gave us all or the best part of their accounts, together with most of the other information asked for in the question schedule. Another 22 firms answered all or most of the questions, but refused to give us details of their accounts. Thus three-fifths of the firms were willing to co-operate up to a point, but less than half the firms gave us adequate details of their balance sheet and trading account. This relatively low response requires some scrutiny. As far as can be ascertained, the refusal rate was about the same in the group of firms with less than 50 and in the group of firms employing more than 150 people, but very much higher in the middle size-group. Of the approximately sixty non co-operating firms about two-thirds refused flatly to admit the interviewers to the manager. In the remaining cases the interviewer gained access but was not allowed to proceed with the interview.

On the surface it would appear that response rates in a business survey need not be lower than those found in a survey of income and savings in the more well-to-do groups or amongst the self-employed. But whereas in a survey of income and savings a single person, normally the head of a household, decides whether or not to give information, in a business survey it very often requires the agreement of several persons to secure information. One in four or five of the refusals was due to the fact that one of the directors was prepared to co-operate fully but could not obtain the agreement of other executives. Divided opinion within the firm about the desirability of revealing financial information almost invariably leads to a final refusal. About one-fifth of the non-respondents said they were too busy just at that time to give information. Another fifth regarded the survey as an intrusion into a sphere that they regarded as strictly private, or believed that to reveal financial information might benefit their rivals. A few firms refused because their accountants advised them not to co-operate. There were, of course, a few cases of illness or prolonged absence of managers; and a variety of other reasons were used to explain refusals.

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[1] Total assets is simply the total amount found on the right-hand side of a balance sheet. Thus it includes financial and non-operational assets in addition to fixed asssets, stocks, etc.

When business men agreed to co-operate in the survey, they often gave us much of their time and took trouble to give accurate and comprehensive data. About half the respondents let us copy accounts on the spot or sent them on loan to the Institute. The other half refused direct access to the accounts, but were quite willing to let us build them up by our question and answer method. Many of these built-up accounts were remarkably accurate, some to the nearest £100, and some even to the nearest pound. To build up accounts this way is more time-consuming for business man and interviewer, but since about half the co-operating businesses preferred this method it should probably be retained in future surveys.

Could the response rates be improved ? It is possible that a slower but more personal approach to the business men, and the backing of the survey by industrial associations or chambers of trade (a backing which may not be obtainable) would raise the response rate; so also might a change in the field-work period to spring or summer. The publication of the methods and results of the pilot survey may also contribute to dispelling distrust among business men about the use we wish to make of the financial data. If we had included firms with under ten employees, as we did in the trial interviews, the response rate might have been greater; on the other hand the reliability of the data might have been inferior, since a good many of the smaller firms do not keep formal balance sheets and profit and loss accounts. There is no easy way of overcoming some of the obstacles to co-operation, especially the difficulty of getting the agreement of several executives before financial data can be released.

In Table III the distribution of firms by industrial classification in the effective sample is compared with that of firms which responded to our survey.

The effect of the elimination of quoted public companies and of firms outside the size range and outside the Birmingham conurbation, was to increase the proportion of firms in the metal-using industries. These (Orders V–VIII)

TABLE III

Sample Distributions by Standard Industrial Classification Orders and Response Rates

Standard Industrial Classification Orders		*Initial Sample*		*Effective Sample*		*Response to Interview*			
						Respondents giving Accounts		*All Respondents*	
		No .of Firms	%	*No. of Firms*	%	*No. of Firms*	%	*No. of Firms*	%
III	Mining Products ...	10	2.2	4	2.8	2	3.1	3	3.4
IV	Chemicals	18	4.0	3	2.1	0	0.0	0	0.0
V	Metal Manufacture ...	30	6.7	11	7.7	5	7.7	9	10.3
VI	Engineering, Shipbuilding and Electrical Goods	54	12.0	22	15.5	10	15.4	11	12.6
VII	Vehicles	54	12.0	18	12.7	6	9.2	8	9.2
VIII	Metal Goods	135	30.0	59	41.5	30	46.2	39	44.8
IX	Precision Instruments, Jewellery, etc. ...	10	2.2	1	0.7	1	1.5	1	1.2
X	Textiles	7	1.6	1	0.7	1	1.5	1	1.2
XI	Leather and Fur ...	5	1.1	1	0.7	0	0.0	0	0.0
XII	Clothing	3	0.7	1	0.7	0	0.0	1	1.2
XIII	Food, Drink and Tobacco	43	9.6	6	4.2	2	3.1	3	3.5
XIV	Wood and Cork ...	19	4.2	4	2.8	1	1.5	2	2.3
XV	Paper and Printing ...	21	4.7	4	2.8	1	1.5	2	2.3
XVI	Other Manufacturing ...	41	9.1	7	4.9	6	9.2	7	8.0
	Total[1]	450	100	142	100	65	100	87	100

[1] With the addition of the sub-group sample (see Table II) the total becomes for the Effective Sample 148, for Respondents giving Accounts 68, and for All Respondents 90.

were originally weighted to constitute 61 per cent of the sample, and came to 78 per cent in the effective sample, and almost the same in the final sample of respondents. Our sampling method has, therefore, not produced an industrial distribution of respondents that matched the estimated actual distribution of Birmingham firms by industry groups. But while the metal-using industries are somewhat over-represented, these are the firms which dominate the Birmingham scene and we may, therefore, expect the Survey to reflect many of the characteristics of small businesses in Birmingham, and some, perhaps, of small firms in general.

II

SOME GENERAL CHARACTERISTICS OF THE SAMPLE

We have already described, in a previous section, how our firms are distributed among industries. We now propose to describe how they are distributed in respect to certain further characteristics such as legal form or constitution, type of ownership and control, age, and size.[1] Even a cursory description, however, of firms cannot be completed by using this type of information alone. It is desirable that something be said about market situation and behaviour. It is difficult to get any statistical information on this subject but in our interview we asked firms for their opinions on it and we shall summarise what they had to say.

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[1] The reader is warned that most of our distributions vary in number of firms because of varying response rates to different questions. In view of the small absolute number of firms in our sample, it was thought unwise to reduce the number of firms in any distribution.

Legal Form. As the following table shows, our survey became in effect a survey of private limited companies. Of the six legal forms shown in the table, limited partnerships and private companies (unlimited) are rare in manufacturing. It is not surprising that we obtained only a few public companies, since by design we were only concerned with those whose shares were not quoted in any of the stock exchanges in Great Britain. We had intended to get quite a number of non-companies (partnerships and sole-ownerships) in our sample, but in cutting out firms employing under 10 we cut out most of the non-companies in manufacturing. This leaves us with private limited companies. The private limited company is a very significant legal form as it is the one adopted by the majority of family businesses in this country. It is a very convenient form in that it provides the benefit of limited liability without the obligation of submitting accounts to the Registrar of Companies.[1] One of the reasons given us for refusal to let us see accounts was based on this very fact. Firms argued that they had become, and intended to remain, private limited companies just because they did not want anybody, even their own employees, to see their accounts. Why, then, should they let us see them ?

TABLE IV

Distribution of Firms by Legal Form

	No. of Firms
Sole Ownership	2
Partnership	5
Limited Partnership	0
Private Company	1
Private Limited Company	74
Public Company (Unquoted)	3
Total	85

We have some information about changes of legal status that have occurred, which is summarised in Table V. Out of the total of 84 firms 49 have changed their legal form at least once, 4 have changed it twice and 1 has changed it three times. The most common change was from partnership to private limited company and the next most common from sole ownership to private limited company. A surprising number of limited companies were once unlimited; the explanation is probably that they were formed before limited liability came to be generally accepted.

Ownership and Control. Private limited companies can have up to 50 shareholders and still remain ' exempt '. But in our sample there were only about 10 per cent of firms with more than 10 shareholders, and three-quarters of

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[1] Except for companies which are described as ' non-exempt private companies '. A private company is ' non-exempt ' if " a body corporate is holder of any shares or debentures, or is beneficially interested in such, or if a director of the company, or any person other than the registered holder has any interest in any of the shares or debentures, or likewise if the number of members or debenture holders exceeds 50 ", A. B. Levy, *Private Corporations and their Control*, Volume 2, page 830.

TABLE V

Changes in Legal Status of Firms (49 Firms)

Present Legal Status	*Original Legal Status*				
	Sole Ownership	*Partnership*	*Limited Partnership*	*Private Co.*	*Public Co.*
Public Company	1[1]	1	0	1[1]	–
Private Limited Company ...	13[2]	20[3]	1	9	1
Partnership	2	—	0	0	0
Total	16	21	1	10	1

[1] These firms changed to Private Ltd. Companies before becoming Public Companies.

[2] Included in this group are *one* firm which became a Limited Partnership and *two* firms which became Partnerships before becoming Private Ltd. Companies.

[3] This group includes *one* firm which changed to a Sole Ownership on the death of one of the partners, then became a Private Co. and finally changed to a Private Ltd. Co.

the firms had six shareholders or less, the mode being three. The small number of shareholders is very largely due to the fact that firms in our size range tend to be predominantly family businesses. Out of 84 firms, more than three-quarters were firms in which one family had a majority interest, and almost two-fifths were firms completely owned by one famly.

The distribution of family businesses by size groups is given in Table VI below. In the smallest size group more than half the businesses are fully owned by one family, but the proportion declines sharply as size increases. Firms in which one family holds at least the majority interest predominate in all size groups. In the smallest size group they constitute four-fifths of all businesses; and even in the largest size-group three-fifths. We did not enquire into the form and dispersion of minority holdings or into the distribution of ownership among families of the 20 non-family businesses. If we had we might well have found that in effect a few more were to all intents and purposes family businesses.

TABLE VI

Distribution of Family Businesses by Size and Family Holding

Size group (total employed)	*No. of firms in size group*	*No. of firms in group in which one family holds :*		
		Full Ownership	*Majority Interest*	*All Directorships*
10—49	46	24	36	27
50—149	28	8	22	8
150—250	10	0	6	4
Total	84	32	64	39

Apart from ownership the family unit can exercise control by holding directorships. In just under half the firms all directorships were held by one family. The number of directors per firm in businesses of our size range is of course small, typically 2 to 4. Only 12 per cent of our firms had more than four directors.

Most of our firms were managed by their owner or principal shareholder in the role of managing director, with generally a works manager or works foreman under him; only one-fifth employed a general manager.

Eight firms were subsidiaries of other firms, nine had subsidiaries of their own; included among these was one firm which was both a subsidiary and had a subsidiary. Five firms had branches and 27 had associated firms. Subsidiaries are awkward to deal with because they are not autonomous financial units. Tactical decision making is devolved in a degree varying with the particular parent company concerned; but the bulk of the finance required by the subsidiary is often in any case provided on loan account by the parent firms, the net profits of the subsidiaries being appropriated to the parent companies and any ploughing back being decided on by the parents. Associated firms present none of these difficulties, but they often help each other financially and this must be borne in mind.

Age of Business. The age structure of our small and medium businesses is given in Table VII which shows the original foundation years of firms.

TABLE VII
Distribution of Firms by Period of Foundation

Period of Foundation	*No. of Firms*
Pre-1919	46
1919—1930	17
1931—1939	7
1940—1945	5
1946—1950	9
1951—1954	1
Total	85

No less than 52 per cent of our firms were founded before the first World War; in fact, 42 per cent were founded before the beginning of the 20th century, 28 per cent before 1875 and nearly 12 per cent before 1850. We even had one firm which was founded sixteen years before Adam Smith published the Wealth of Nations.[1] Naturally the nature of the products manufactured by many of the firms may have changed partly or completely since foundation; product variation, of course, is not uncommon, even during quite short periods, and is a part of the adaptation process that makes for survival. Almost one-eighth of the sample firms were founded since the war. which, if it were typical of industry as a whole, would appear to indicate quite a high rate of new entry. Further information regarding age structure was obtained by asking for the date when the present owners took over. Ownership, however, is an extremely ambiguous concept. If a son takes over a proprietorship from his father, or if a new partner is brought into the business —not to say if a family partnership becomes a private limited company— has ownership changed ? While it is possible to resolve such difficulties in principle given all the relevant facts, it is very doubtful if we made clear to our firms exactly what we meant by change of ownership.[2] In most cases the dates given for the change in ownership are the same as those given for the change in legal form. The distribution of firms by commencement of present ownership is given in Table VIII.

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[1] Unfortunately it is not a pin manufacturer.

[2] We instructed our interviewers that we only wanted to record a change where the majority interest changed.

TABLE VIII

Distribution of Firms by Commencement of Present Ownership

Period of Commencement	*No.of Firms*	%
Pre-1919	14	16.7
1919—1930	14	16.7
1931—1939	17	20.2
1940—1945	14	16.7
1946—1950	17	20.2
1951—1954	8	9.5
Total	84	100

A two-way breakdown of firms by size (measured in terms of employment) and by age is given in Table IX. There is no close relationship between size and age of firms, but the majority of old firms have more than 50 employees while most of the new firms (post 1919) are small ones. Or, to look at it in a different way, of the 38 firms with more than 50 employees 74 per cent had been established before 1919. Of the 46 firms with under 50 employees about a third were of pre-1919 vintage, about a third dated from the inter-war years and about a third were young firms.

TABLE IX

Distribution of Firms by Size and by Age

Foundation of Firm	*10–49 No. of Firms*	*50–149 No. of Firms*	*150–250 No. of Firms*	*10–250* No. of Firms	*10–250* %
Pre-1919	17	23	5	45	53.6
1919—1938	15	4	4	23	27.4
1939—1954	14	1	1	16	19.0
Total	46	28	10	84	100

Market Structure. At the best of times it is difficult to measure market structure. Modern analysis has shown that the numbers of firms in an industry is an insufficient guide to the degree of monopoly or competition ruling therein, as this is in part a function of the extent of product differentiation. In principle the appropriate measure of the degree of competition facing a firm would be framed in terms of certain elasticities and cross-elasticities of demand and supply,[1] but in practice such a measure is very difficult, if not impossible, to obtain. Nevertheless we considered it essential to make some attempt to elucidate the market position and behaviour of our firms. For lack of a better alternative we sought information regarding market structure by asking for the views on it of the firms themselves. These should be treated with some reserve. It was inevitable, for practical reasons, that we should put our questions in a general way, using terms such as ' competitive ' or ' highly competitive ' without precise definition. There is, however, a danger in such a procedure that different people will attach a different meaning to the same terms and that their interpretations of a question and answers to it will vary accordingly, although answers which only give general opinions may be partially checked by careful analysis of profit rates and margins and other similar data.

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[1] See Robert L. Bishop, ' Elasticities, Cross-elasticities and Market Relationships ', American Economic Review, Vol. XLII, December 1952, together with the references contained there.

Out of a total of 85 firms nearly 90 per cent said that they thought that the market in their main line of business was highly competitive. This on any account is a high proportion and needs some discussion. First, however, let us examine the nine firms who said that their markets were not competitive. All of them are in the metal using industries—S.I.C. Orders V to VIII; but this is not particularly significant as most of our population are in this group. There appears to be no significant difference between the ages of the nine and those of the rest of our firms, if age is reckoned from the date of foundation; but if it is reckoned from the date at which present owners assumed control our nine appear on the whole to be somewhat younger. Six out of the nine firms were members of their trade association (one firm was not ascertained) but none of the trade associations was said to fix prices. Of the nine, five sold mainly in the local market (one not ascertained). But what is most interesting about them are the reasons they gave for the absence of competition in their markets. Two said that they were monopolies because their superior ' design ' had got them out of competitive positions; one said it was a patent monopoly; one, selling nationally had only one competitor; yet another was 'one of a few'; two claimed that their trades were highly specialised; one stated its market was not highly competitive at the moment because trade was good; and finally the ninth said it just did not go into competitive markets.

What then are the factors leading to the alleged highly competitive position of the other 76 firms ? Evidently the firms themselves do not believe competitiveness merely a function of numbers in the industry : out of a total of 85 firms 90 per cent regarded their markets as highly competitive, while only 46 per cent said they had a large number of competitors, 37 per cent saying they had a few and the remaining 17 per cent saying they had a moderate number. These same firms were then asked if they could state approximately the number of their competitors. The resulting distribution (Table XI) shows wide dispersion, but just under 50 per cent considered that they had between 10 and 50 competitors. Fifteen firms which said they had under 10 competitors were tested for similarity of product, age, size, growth and kind of market, but no significant uniformities appeared.

We went on to ask the firms whether they thought their markets had become more competitive in recent years or less competitive or had remained about the same. Out of a total of 84 firms, 70 per cent said ' more competitive ', 22 per cent said ' about the same ' and only 8 per cent said ' less competitive '. Among the ' more competitive ' firms about a fifth cited as the reason for the increase in competition the easing of raw material supplies

TABLE X

Distribution of Firms by Number of Competitors

No. of Competitors	*No. of Firms*	%
1	1	1.4
2	3	4.2
3	1	1.4
4	2	2.8
5—9	8	11.3
10—19	16	22.5
20—49	17	23.9
50—99	5	7.0
100—499	4	5.6
500—999	2	2.8
1000 and over	2	2.8
'Hundreds' or 'Too many to say'	10	14.1
Total	71	100

and about a quarter the end of the seller's market; the remainder gave reasons so various that no useful purpose would be served in trying to classify them. Among the 'less competitive' firms, one attributed the decline in competition to a reduction in the number of competitors, and another to a technical development which had put him right in front of his erstwhile competitors; the other five gave various reasons hard to understand.

Certain firms which sold in geographically separate markets faced a different degree of competition in each. Very often, for example, competition would be slight in the local market[1] but intensive in more distant areas. Accordingly our firms were asked to give an estimate of their position in the chief market for their main line of business. Out of a total of 85 firms only eight firms sold *entirely* in the local and only 11 *entirely* in the national market. The majority of firms (42) sold *entirely or mainly* in the national market, while 31 firms sold *entirely or mainly* in the local market and only three in the export market. The firms' views regarding their competitive position were set against their market background, but no significant relationship emerged. Thus of the nine firms saying their market was not high.y competitive, four sold mainly in the local, three in the national, and one in the export market, with one not ascertained. Moreover, the five firms which attributed the absence of competition to their possession of a monopoly or to the fact that only a few firms sold the same product were more or less evenly distributed between local and national markets.

A very high proportion of firms sold mainly to 'other manufacturers'. This is accounted for partly by the size range of our sample and partly by the fact that we did our Survey in Birmingham. Many of our firms were serving larger firms, and were, in effect, sub-contractors. Quite a high proportion—27 per cent—were selling to National or Local Government authorities, including nationalised industries.

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[1] Which we defined as in and around Birmingham.

TABLE XI

Distribution of Sales Outlets (81 Firms)

Selling to :—	*No of Firms*[1]
Other Manufacturers	74
Wholesalers and Retailers	44
Public direct	9
Others (mainly Government and Nationalised Industries, etc.)	23

[1] Thirty-four Firms sold to Other Manufacturers only and 10 to Wholesalers and Retailers only. Three Firms sold to Other Manufacturers and Wholesalers and Retailers ; 8 to Other Manufacturers and the Public direct ; 23 to Other Manufacturers and ' Others ' ; 5 sold to Wholesalers and Retailers and the Public direct ; and 11 to Wholesalers and Retailers and ' Others '.

Next, we tried to find out more about the forms of competition used in firms' markets. Again, *faute de mieux*, we asked opinion questions. Table XII shows the frequencies with which different forms of competition were cited in the firms' answers.

TABLE XII

Forms of Competition in Highly Competitive Trades (76 Firms)

Forms of Competition	*No. of Firms*	%
Price	70	92.1
Quality	22	28.9
Delivery dates	12	15.8
Advertising	21	27.6
Salesmen	32	42.1
Personal contact with customers	52	68.4
Product changes	15	19.7
Other	3	3.9

The frequency value for each form of competition may well be exaggerated, especially that for ' price ' competition. Nevertheless, 92 per cent of the firms interviewed said they faced price competition, and most of them regarded it as the most prevalent and compelling form of competition. A strong runner-up was competition by ' personal contact with customers '. The number of firms was too small to show whether different kinds of firms or different industries compete mainly in different ways. Since our firms are mainly small and sell mostly to other manufacturers it is not surprising that only a fifth said that advertising was used as a form of competition in their markets. Eighty-two per cent, however, said they used some form of advertising : most used trade journals as the following table shows.

TABLE XIII

Forms of Advertising (85 Firms)

	No. of Firms	%
Press	17	20.0
Trade Journals, Directories, etc.	52	61.2
Hoardings, Posters, Buses, etc.	14	16.5
Mail Leaflets	15	17.6
Other	2	2.4

Although just over 60 per cent of the firms advertised in trade journals, the absolute amounts spent were very small. Total selling costs were also small generally, although less so in the case of firms employing travellers and other outside salesmen.

Trade associations do not appear to affect the market position of firms in our size range. Out of a total of 77 firms answering the relevant question, 65 per cent were members of an association, but some thought associations not very beneficial or even positively harmful, and some regarded their influence as more formal than real. For example, one business man said that he absolutely refused to join the trade association in his industry because he was against price fixing of any kind. Another spoke bitterly against his trade association for fixing prices because, he claimed, they were fixed too high and spoilt his export market. A third said that he had withdrawn recently from his association because he could not see any benefit in belonging to it. However, their views cannot be said to be typical, as the majority of firms did not volunteer any opinion.

Of 49 firms belonging to trade associations and answering the relevant question, just under 30 per cent said that their prices were ' regulated ' by their trade association.[1] No specific questions were asked about the various ways in which prices may be fixed, but one firm said its association fixed only quantity rebates, while another specified cash discounts, and a third retail profit margins. Two firms stated that their associations could not fix prices because of the nature of their products.

Out of a total of 79 firms answering the question ' Do you think trade associations help to keep prices stable ? ' 29 (37 per cent) said ' Yes ', 37 (47 per cent) said ' No ' and 13 (16 per cent) said ' We do not know '.

From all these facts it would appear that small and medium-sized businesses are probably not nearly so much in favour of trade associations and their controlling activities as is often believed.

The distributions of firms by number of customers is given in Table XIV. The four firms having under 10 customers were all young and small, and two

TABLE XIV

Distribution of Firms by Number of Customers

No. of Customers	*No. of Firms*	%
Under 5	3	4·2
5—9	1	1.4
10—19...	6	8.3
20—49...	10	13.9
50—99...	9	12.5
100—499	23	31.9
500—999	6	8.3
1000 and over	8	11.1
' Hundreds '	6	8.3
Total	72	100

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[1] In only one case was a sales quota reported to be in operation.

were declining firms. These four firms appeared otherwise dissimilar. For all firms there was some positive correlation between number of customers and size and between number of customers and age.

Almost all firms claimed that they dealt regularly with most of their customers. In some cases the nature of the business and other evidence cast doubts on the validity of the claim, and perhaps this is one instance where inadvertently we asked a leading question to which the business man felt he ought to answer in the positive.

The Survey questions provide a very incomplete picture of the market situation facing individual firms and of the market structure of an industry, although if the intensive sampling of a fairly homogeneous industrial subgroup had been successful we might have obtained views of one market as seen through the eyes of firms in the same industry with different cost structures. Nevertheless the Survey does bring to light—and in this respect there could be no substitute for a survey—certain market characteristics and attitudes of a selection of small businesses. If a survey does not help to answer long-standing questions—and this is the more probable if it is only a pilot survey—it may at least raise some new ones. It is sometimes said, for example, that price competition is on the decline. Why, then, did so many of our firms stress the importance of price competition, and how are their assertions about this to be understood ? As we have not collected details of pricing method, price changes and price differentials, we cannot even try to answer these questions. But the fact that business men in our sample assert that price is an important element in their competitive situation remains of considerable interest.

Size Distributions. There is no unique sense in which one firm can be said to be ' bigger ' than another. We may take ' bigger ' to mean, for example, ' having a greater employment ', or ' having a greater quantity of capital employed ', or ' producing a greater output '. Accordingly if we are to give a reasonable account of how our firms vary in ' size ', we must give independent consideration to different characteristics, like the ones quoted, of labour employed, capital employed, and output; and with these three, in fact, we shall mainly concern ourselves in the present section.

When, however we try to obtain practical measures of the above mentioned characteristics certain snags arise. For example, as a measure of labour employed, should we take only the number of manual labourers employed, or should we include non-manual labour, such as foremen, office and technical staff ? Again, measuring output has its complications. Output, in the sense of value added, is only obtainable from fully detailed accounting data. A crude indicator of output that is available, for almost all other firms, is turnover, or net sales. But this is subject to the objection that it reflects input values, the ratio of which to output, may vary considerably from firm to firm. Moreover, if the value of outputs of different firms are being compared over a number of years they should all, in principle, be deflated by appropriate price indices, which would be a formidable task. It is similarly difficult to find specific accounting figures to represent ‘ capital employed ’. Possible measures, for example, are total assets, fixed assets, or just plant and equipment. Total assets are probably the most useful for our purposes. But if, say, one were trying to calculate ‘ capital intensity ’, one might take fixed assets alone, and would almost certainly want to deduct from total assets any holdings of non-operational assets. Where some firms own their premises while others rent them there may be an argument for basing comparisons on total assets less land and buildings.

Some compromise must inevitably be made with what is practicable. For measuring size in the sense of numbers employed we shall use figures for total employment including both manual and non-manual workers, for capital employed we shall use total assets, and for output we shall use figures of turnover or net sales.

The distributions of firms by size characteristics are given in Table XV. It will be noted that the total number of firms in each size distribution varies : Eighty-four firms gave us their total employment, but of these 6 did not disclose their turnover and 19 gave no total asset figures.

Table XV

Distribution of Firms by Three Size Characteristics

Empl. group	*No. of Firms*	*%*	*Net Sales £000*	*No. of Firms*	*%*	*Total assets £000*	*No. of Firms*	*%*
10– 19	13	15.5	Under 10	2	2.6	Under 5	6	9.2
20– 29	13	15.5	10 and under 20	14	17.9	5 and under 10	9	13.8
30– 39	11	13.1	20 „ 30	8	10.3	10 „ 20	10	15.4
40– 49	9	10.7	30 „ 40	6	7.7	20 „ 30	6	9.2
50– 59	6	7.1	40 „ 50	7	9.0	30 „ 40	5	7.7
60– 69	5	6.0	50 „ 75	8	10.3	40 „ 50	2	3.1
70– 79	4	4.8	75 „ 100	7	9.0	50 „ 75	7	10.8
80– 89	5	6.0	100 „ 150	13	16.7	75 „ 100	3	4.6
90– 99	2	2.4	150 „ 200	2	2.6	100 „ 150	4	6.2
100–149	6	7.1	200 „ 250	3	3.8	150 „ 200	3	4.6
150–199	7	8.3	250 „ 500	5	6.4	200 „ 250	5	7.7
200–250	3	3.6	500 and over	3	3.8	250 and over	5	7.7
	84	100		78	100		65	100

The mean, the median and the quartiles for the three distributions are set out in Table XVI. Half our firms employed less than 50 people, half had a turnover of less than about £50,000, and half total assets of less than

Table XVI

Summary of Size Distributions

	Employment	*Net Sales*	*Total Assets*
Mean	64	£101,000	£86,000
1st quartile	24	£24,000	£11,000
Median	45	£51,000	£34,000
3rd quartile	80	£120,000	£106,000
No. of firms	85	78	65

£40,000. The three distributions are all skewed, with the arithmetic means higher and nearer the third quartile than either the median or the mode.

It is of some interest, since we have basic data for the three size characteristics, to go on and see how the ratio of turnover to employment, which is a crude sort of productivity ratio, and the ratio of total assets to employment, which is a measure of capital intensity, vary from firm to firm. The distributions of these two ratios are given in Figures 1 and 2. Although most of our sample belong to the metal-using group, there is little homogeneity among such firms, and we should expect, accordingly, to find considerable variability in both capital intensity and productivity ratios. The first ratio shows a more regular pattern than the second: this is understandable since the second reflects, among other things, differential input values.

Despite the lack of homogeneity of our sample, it is also of some interest to explore whether there is any connection between size of firm and (*a*) labour productivity, (*b*) capital intensity, (*c*) output capital ratio in the sense of sales or value added per £ of assets. It would be interesting to see whether the three ratios themselves are interrelated and to ask, for instance, whether labour productivity increases and the output capital ratio falls because capital per man increases with size. We have only made a beginning at this, and therefore can only report provisional findings in a general way, that is without giving computational details, if only because more work has to be done before quantitive conclusions can be drawn.

From scatter diagrams it would seem that sales per man tend to increase with size, whatever size characteristic we use. A slight negative relationship exists, on the other hand, between sales per £ of total assets and size in the sense of total assets or sales. The following table illustrates the decline in the ratio of sales to assets for different asset size groups.

Table XVII

Average Net Sales/Total Assets by Asset Size Groups

Total Assets	*No. of Firms*	*Ratio of Net Sales to Total Assets (Average for Group)*
Under £40,000	36	2.6
£40,000—£149,999	14	1.7
£150,000 and over	9	1.5
	59	2.2

Capital per man does not seem to increase with size if employment and sales are used as size criteria, but it does appear to some have positive association with asset size. Firms having larger sales and employment are generally relatively capital intensive, but the capital intensity of the smaller firms is widely dispersed between relatively high and relatively low values.

Substituting value added for net sales, and fixed assets for total assets, we find that value added per man is again a function of size however measured, and that fixed assets per man also varies positively with employment and asset

Assets and Sales per Man Employed

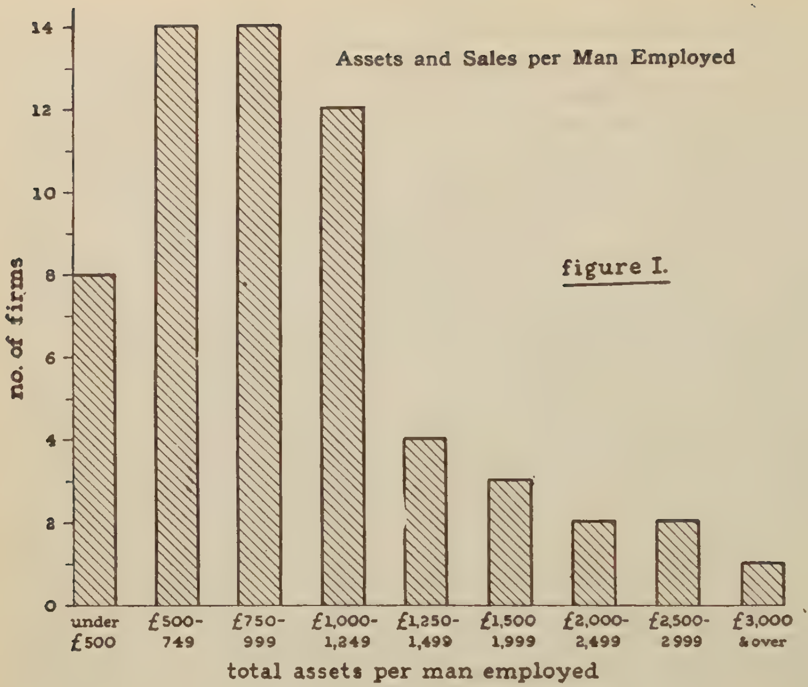
figure I.

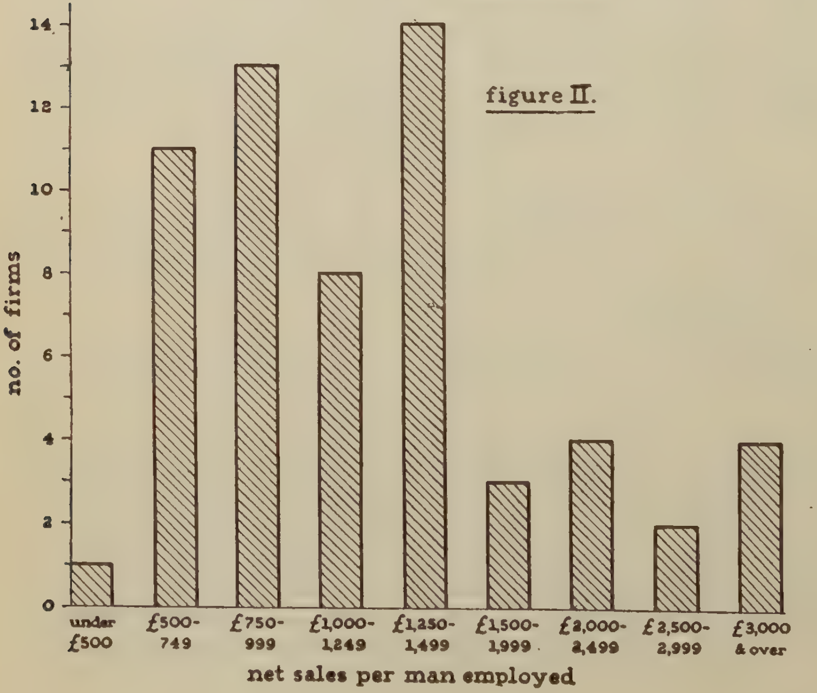
figure II.

size. The association between value added per £ of assets and size is again not very clear, and it is not made clearer by the elimination from the scatter diagram of firms which rent their premises.

The above analysis is obviously inconclusive. The sample is too small to provide fairly homogeneous groups for a breakdown by size characteristics and for other refinements we would like to introduce, but if some of our subsequent analysis yields firmer results we may return to this matter in a later article

Profitability. The description of our firms would be incomplete without at least a provisional report about their profitability. Table XVIII gives the distribution of firms by rate of gross profit[1] on total assets and by employment size groups.

TABLE XVIII

Distribution of Firms by Rate of Gross Profit or Loss on Total Assets and by Employment Size Groups, 1953

Total Employment	*% Profit*						*% Loss*			*Total*
	Over 30	*20–29*	*15–19*	*10–14*	*5–9*	*0–4*	*0–4*	*5–9*	*Over 10*	
10—29 ...	1	—	3	2	2	5	1	2	1	17
30—49 ...	2	1	1	3	1	2	—	2	—	12
50—69 ...	—	1	2	—	2	2	—	—	—	7
70—99 ...	—	1	1	1	3	—	1	—	—	7
100—150...	—	1	2	1	1	—	—	—	—	5
Over 150	—	1	1	1	1	3	—	—	—	7
Total	3	5	10	8	10	12	2	4	1	55

The outstanding feature of this table is the wide dispersion of profit rates within size groups and the concentration of loss firms in the smallest size groups. Out of 29 firms with less than 50 employees about one-fifth made losses, whereas of the 26 firms employing more than 50 people only one made a loss. At the other end of the scale small firms are again conspicuous. Of the 18 firms with profit rates of more than 15 per cent as many as 45 per cent had less than 50 employees. Generally speaking the larger firms are making high rates of profit, and one might guess, though it cannot be shown from our data, that they are making good profits more consistently.

The same sort of picture is presented if assets or sales are used for size classification, or if net instead of gross profits are taken, or if the profit rates are calculated on net worth instead of total assets. Table XIX below gives gross and net profit rates on total assets and net worth without a breakdown of firms by size.

About 11 per cent of the firms made losses; as we have seen already these were mainly smaller firms. Almost a third of the firms earned more than 15 per cent gross on total assets, and almost three-fifths did so on net worth. The net profit distributions are much more bunched. In fact, more than four-fifths of the firms had a net profit on assets of under 15 per cent; very

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[1] For a definition of the term see p. 249.

Table XIX

Distribution of Firms by Various Profit Rates, 1953

	Gross Profit as % of:				*Net Profit as % of:*			
	Total Assets		*Net Worth*		*Total Assets*		*Net Worth*	
	No. of firms	*% of firms*	*No. of firms*	*% of firms*	*No. of firms*	*% of firms*	*No. of firms*	*% of firms*
(Profit rates) :								
50 and over	—	—	4	7.1	—	—	1	1.8
30—49	3	4.9	10	17.9	—	—	1	1.8
20—29	6	9.8	9	16.1	2	3.3	6	10.7
15—19	10	16.4	9	16.1	1	1.7	7	12.5
10—14	12	19.7	6	10.7	6	10.0	8	14.3
5—9	11	18.0	4	7.1	22	36.7	15	26.8
0—4	12	19.7	8	14.3	22	36.7	12	21.4
(Loss rates)								
1—4	2	3.3	2	3.6	2	3.3	2	3.6
5—9	4	6.6	—	—	4	6.7	—	—
10 and over	1	1.6	4	7.1	1	1.7	4	7.1
Total	61	100	56	100	60	100	56	100

few indeed had rates of more than 15 per cent. This seems to fit in with the general belief that profit rates on total assets are only moderately high. It should be noted that almost half the firms have a net profit rate on capital employed of less than 5 per cent, which is sometimes taken as a minimum return. The picture is altered when we take net profits on net worth : more than two-fifths of the firms have rates in excess of 10 per cent and more than a quarter rates above 15 per cent.

Table XX

Distributions of Firms by Rates of Retained Profits on Total Assets and Net Worth, 1953

	Retained Profits as % of Total Assets		*Retained Profits as % of Net Worth*	
	No. of firms	*% of firms*	*No. of firms*	*% of firms*
20—60	0	0.0	5	8.9
15—19	2	3.3	3	5.4
10—14	4	6.7	6	10.7
5—9	17	28.3	15	26.8
1—4	25	41.7	17	30.4
0	5	8.3	3	5.4
Loss	7	11.7	7	12.5
Total ...	60	100	56	100

How much of the net profit was ploughed back into the business ? Table XX, giving retained profits as a percentage of total assets and net worth, shows that about one-fifth of the firms did not or could not retain profits, while about two-fifths ploughed back profits up to 5 per cent of the capital employed, and almost two-fifths managed to increase their assets by 5 to 20 per cent by ploughing back profits. The additions to net worth out of retained profits are, as the table shows, even more striking. Half the firms retained profits of more than 5 per cent of their net worth and one-sixth more than 15 per cent.

Indices of Recent Growth. It is clear that most of the firms in our sample which are relatively large at present must have grown, but by how much we do not know. Moreover, our sample cannot, of course, reveal anything about mortality rates, or what number of firms have grown beyond the size range covered, or have been turned into public companies. Consequently, we cannot deduce anything definite about rates of growth over the whole period. However, we do at least know (Table XXI) that nearly 90 per cent of the firms founded during the sixteen years from 1939 to 1954 had under 50 employees. If we take those founded in the inter-war period, still 65 per cent were under 50. Finally, it is remarkable that nearly 40 per cent of the firms founded prior to 1919 had under 50 employees. This can be put another way: of the 45 firms founded before 1919 only 5, or just over 10 per cent, had grown into the 150—249 employment class. Also, only 1 out of the 16 firms founded since 1939 has managed to get into the top size group.

TABLE XXI

Distribution of Firms by Age and Size

Total Employment	*Foundation Period*						*Total*	
	Pre-1919		*1919–1938*		*1939–1954*			
	No.	*%*	*No.*	*%*	*No.*	*%*	*No.*	*%*
Under 50 ...	17	37.8	15	65.2	14	87.5	46	54.8
50—149 ...	23	51.1	4	17.4	1	6.3	28	33.3
150—250 ...	5	11.1	4	17.4	1	6.3	10	11.9
Total ...	45	100	23	100	16	100	84	100

Turning now to more recent growth, we have fairly accurate figures from which to calculate growth rates for employment and turnover for the period 1947 to 1953, but no detailed accounts for all these years. We also asked subsidiary questions about the ‘ Value of main raw materials used ’, ‘ Fixed assets ’ and ‘ Either current or total assets ’ for the three years 1947, 1950 and 1953. Some firms went to great trouble to look up their figures for us, but some just guessed the answers and others refused to give information; accordingly we have doubts about using the data obtained. But we shall make use of the answers which businessmen gave about changes in productive capacity and capital utilisation. Rates of growth will be given as average annual percentage changes in employment and turnover between 1947 and 1953. In calculating the rate of change over time it is sometimes preferable to use the exponential rate, relating each year's change to the size at the beginning of the year and not to the original size. This has the advantage of narrowing a range of widely spread rates, but the disadvantage of making interpretation less straightforward, especially if positive and negative rates are involved. Although in later sections we shall employ exponential rates, the tables in this section will give crude annual average rates of change. But it should be remembered that it is not only rates of growth that matter but also absolute growth. A doubling in size of a single large firm, say from 100 to 200, has the same effect on aggregate employment as a six-fold increase, say from 2 to 12, in ten small firms. We shall therefore also give the absolute change for four size groups of firms.

Table XXII summarises the information about growth rates, and gives in columns (*a*) the distribution of all the firms which supplied us with the relevant information; in columns (*b*) the distribution of the smaller number of firms that gave both employment and turnover figures for the years 1947 and 1953. Its most striking feature is the high proportion of firms that have not grown or have actually declined during the six-year period. Taking turnover first, it can be seen that about 15 per cent of the firms had declined, about 13 per cent remained stationary, and 73 per cent had grown, in greatly varying degrees. But if we take into account the increase in prices over the period, the number of firms which declined or remained stationary was in fact much greater than that suggested by the value figures. If the average rate of increase in the prices of mechanical engineering goods over the period was, as it appears it might have been, about 8 to 9 per cent per annum, and if this is roughly representative of the change in prices of our firms' products, then only about 40 to 50 per cent of the firms have really increased their turnover. If growth is measured in terms of employment, the proportion of expanding firms is about 55 per cent, while 17 per cent declined, and more than a quarter reported that their employment had not changed over the period. It seems remarkable that in a period of general expansion of trade and of overall increase in the labour force, only just over half the firms in our sample have increased their employment; and it seems even more remarkable, on the assumption that some productivity-increasing investment occurred during the period, that only about the same proportion, expanded in terms of turnover deflated for price changes.

TABLE XXII

Distribution of Firms by Average Annual Rates of Change in Size 1947 to 1953

Annual average % change	*Employment*				*Turnover*			
	(*a*)		(*b*)		(*a*)		(*b*)	
	No.	%	*No.*	%	*No.*	%	*No.*	%
−10 — −20	1	1.4	1	1.9	0		0	
−5 — − 9 ...	5	7.0	4	7.5	4	7.3	4	7.5
−1 — − 4 ...	5	7.0	4	7.5	4	7.3	4	7.5
0	22	31.0	14	26.4	7	12.7	7	13.2
1 — 4	9	12.7	7	13.2	7	12.7	7	13.2
5 — 9	12	16.9	9	17.0	9	16.4	9	17.0
10 — 14	6	8.5	5	9.4	9	16.4	8	15.1
15 — 24	3	4.2	3	5.7	6	10.9	5	9.4
25 — 49	4	5.6	2	3.8	5	9.1	5	9.4
50 — 99	3	4.2	3	5.7	2	3.6	2	3.8
100 — 250	1	1.4	1	1.9	2	3.6	2	3.8
Total	71	100	53	100	55	100	53	100

Differences in the rate of growth are equally remarkable. The turnover of about a third of our firms rose at annual compound rates of more than 10 per cent, and more than a fifth of the firms expanded their employment at such rates. The very high rates of growth, of course, mostly occur in very small firms (under 20 employees) that grew rapidly from one- or two-man businesses to their present size of less than 20.

The more rapid rate of expansion of the average smaller firm is borne out by the following table which relates to the firms for which we have been able to obtain data for the full six-year period. It gives total employment at the beginning and end of the period and the annual average percentage change in employment for four size groups of these firms.

TABLE XXIII
Changes in Total Employment by Size Groups of Firms, 1947–1953

Size of firm: No. of employees in 1947	*No. of firms in size group*	*Aggregate employment 1947*	*1953*	*Average annual percentage change*
Under 20	23	232	364	9.5
20—49	24	782	884	2.2
50—99	18	1,077	1,248	2.7
100—250...	6	778	872	2.0
Total	71	2,869	3,368	2.9

The smallest firms as a group have grown at an annual rate of almost 10 per cent, four to five times as fast as the bigger ones, which all had growth rates of between 2 and 3 per cent; but the rapid relative expansion of the smallest firms made only a moderate contribution to the aggregate rise in employment of all firms. In fact, the smaller percentage growth of the size group 50–100 added more to the total employment roll than the swift expansion of the small firms, which accounted for about a quarter of the aggregate increase in employment.

We turn now to our third measure of growth, the changes in productive capacity. In asking questions about capacity our interviewers were instructed to stress that we meant the physical capacity of plant and equipment, and not the change in turnover or employment. But we did not probe deeply into the question of how the businessman in his own mind assessed capacity for a normal working week. The answers we obtained are summarised in Table XXIV. No less than 75 per cent of the businessmen said that their firm's

TABLE XXIV
Distribution of Firms by Change in Plant Capacity, 1947–1953

Percentage change	*Number of Firms*	%
Declined	1	1.3
No change	20	25.3
1—9	2	2.5
10—19	10	12.7
20—49	18	22.8
50—99	6	7.6
100—199	15	19.0
200 and over	7	8.8
Total	79[1]	100

[1] Four additional firms had increased their capacity, although they did not know by how much.

productive capacity had increased, while a quarter stated that there had been no change in capacity and one firm reported a decline. Most of the growing firms had expanded their capacity by more than 20 per cent during the six-year period, equivalent to an annual increase of more than 3 per cent. Most of the large capacity changes (as stated by the business men) occurred in the smaller firms..

It would seem that capacity has grown more than employment. This may, of course, be partly due to a substitution of capital for labour, but it should be noted that the deflated turnover figures only show approximately the same proportion of firms growing as do the employment figures. The acute labour shortage in Birmingham, or a deliberate policy of building up some reserve capacity, may be explanatory factors. But there is one further factor that may help to explain the difference, namely the degree of capacity utilisation. We asked businessmen to estimate the degree of utilisation of their plant and equipment at the time of the interview, and their answers are tabulated below. Looking at the over-all picture first, it will be noted that only two-fifths of the firms were stretched to full capacity working. Almost an equal proportion was, however, heavily under-utilised, with unused capacity of from 20 to 50 per cent. That appears to be a high proportion of severe under-utilisation, even if allowance is made for seasonal variations and for the fact that 1953–4 was a year in which at least some of the firms were still recovering from the mild recession in 1952. We have no data to compare capacity used over time, and perhaps it should not surprise us that at any given moment of time there is a considerable slack unevenly distributed in the system. Though under-employment of equipment exists in all size groups, it was slightly higher in the lowest groups than in the higher ones. The proportion of firms with full capacity utilisation is just over two-fifths in all size groups.

TABLE XXV

Distribution of Firms by Plant Capacity in Use and by Size of Firm, 1953

Capacity in use %	*Total employment* 10–49	50–149	150 and over	*All firms* No.	%
Under 60	6	6	0	12	14.6
60—79	12	4	4	20	24.4
80—99	7	6	2	15	18.3
100	18	12	5	35	42.7
Total	43	28	11	82	100

Even after allowing for wider error margins in the businessmen's estimates of capacity and in our own deflator for turnover than in the employment figures, the relationship between the three growth indices is puzzling. It might be accepted as plausible that a greater proportion of firms report capacity expansion than employment increases, but it is much less obvious that—assuming gains in productivity—the proportion of firms showing an increase in real turnover does not exceed substantially that of firms showing an employment increase. In the latter two distributions the proportion of firms that have either not changed in size or have declined is about 40 per cent, but 70 per cent of all firms stated that they were satisfied with thier development, and among this contented group were many which had remained stationary.

III

FINANCE AND EXPANSION

In this section we shall try to relate some of the factors associated with the expansion of the firm to the sources of finance which have been used. A full analysis of profits and other sources of finance will have to wait on a later article, but in the meantime, with the data which have already been analysed, we can show some of the broad relationships existing between the firms' level of profits and the record of their past growth and attitudes to future expansion. It would be as well, perhaps, to stress once more that we are dealing with a very small sample, so that such conclusions as we reach must be extremely tentative and are to be regarded primarily as an indication of the sort of hypothesis that might be tested in a large national sample.

Collection of Data. We enquired about the sources of finance and the quantities used by the firm from the end of the war up to 1953. The balance sheets[1] for each year of the period would have given us a virtually complete picture of the finance used. But, for reasons given earlier, the interview was rather long and we therefore decided to sketch the financial scene over the whole period by questions, and to ask for balance sheets for only the last two years. Thus this section will be in part based on the detailed and complete information contained in those balance sheets which we obtained for 1952 and 1953, but most of the analysis will be based on the answers to our specific questions about finance since 1947. While almost all the firms which permitted an interview answered our questions about finance, only about 70 per cent gave accounts; consequently, when we use data such as profit rates which can only be got from the detailed accounts, we shall be referring to only this percentage of firms. The questions which we asked the businessmen about the period 1947–53 were aimed at getting the following information: the amount of new capital raised in any form, e.g. by issue of shares, debentures etc., or, as with non-corporate business, by the transfer to the business of the owner's personal capital or of the capital brought in by a new partner; the amount of profit retained in the business each year; the amount of loan finance raised over the period (by other means than issue of debentures) and its sources.

This was a rather formidable section, and we had doubts about just what the businessmen could easily, or would willingly, answer. In the event we were surprised by the number of questions which were answered

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[1] Together with information about finance raised and repaid, in part or whole, between balance sheet dates, e.g. overdrafts.

(and quite accurately, we discovered, whenever we could check against balance sheets). It appears that most loans or capital issued in a small firm's recent history stand out as rather uncommon events so that they are easily remembered. But bank overdrafts are harder to pin down since they frequently fluctuate a great deal over the year: while a third of our firms gave us no figures for their overdrafts, the other two-thirds gave what they called ' an average level '. The question on retained profits was least successful. Most firms merely stated that they could not remember the amount; and our interviewers had been instructed not to press too hard for any details which would involve checking back over many years' accounts. Thus, through our questions we obtained a fairly good idea of the part played by loan finance of all kinds and the raising of new capital through shares etc., but for the amount of profits retained in the business we are forced to rely on the accounts for 1952 and 1953 only.

Sources of Finance. Eighty-two firms gave details of their finance over the period 1947–53. All of them ploughed back profits at some time over the period, while 24 of them relied on profits alone and used no other source. In Table XXVI the number of firms using each ' external '[1] source of finance is shown. The number of firms using external sources was 58, just over 70 per cent of the total, but, as can be seen from the table, many firms used more than one source.

TABLE XXVI

Sources of Finance other than Profits, 1947–53 (58 firms)

Source		*No. of firms using source*
New issues of shares		7
New issues of debentures		1
Increase of partner's capital		1
Loans from:	Directors	25
	Partners	1
	Parent, subsidiary, or associated firms	5
	Bank (overdrafts)	35
	Building societies (mortgages)	3
	Other financial institutions (mainly for hire-purchase)	9
	Individuals	1
	Unspecified sources	2

Only two sources, directors' loans and overdrafts, were used by a high proportion of the firms. Two of the seven firms issuing new shares raised the money entirely from new shareholders; two raised part of it from new shareholders, and the remaining three raised all of it from existing shareholders. Much of what we have called external finance was raised entirely from persons or firms already connected with the business; much of it, that is to say, came from new capital provided by existing owners, or from directors' and partners' loans, or from loans by parent, subsidiary or associated firms. Just over half of our 82 firms raised *all* their finance either from profits or from some person or firm already connected with the business; while 39 used some other sources and among these 35 used overdrafts.[1] It would seem, from our sample, that small firms, with very few exceptions, either finance themselves from their private resources (from profits, or through persons or firms connected with the business) or borrow on overdraft from the banks.

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[1] Throughout this section ' external ' finance will mean a source of finance other than profits retained in the business, which we designate as 'internal finance '. Profits are the only strictly ' internal ' source in the sense of ' belonging to the business ', but there is a sense in which directors' loans are more ' internal ' than, say, bank overdrafts. However, our present contrast between ' internal ' and ' external ' finance is of some economic significance and in this article we shall base most of our analysis on it. We shall discuss the distinctions between different ' external ' sources in a later article.

Attitudes to the Use of External Finance. For the rest of this section we shall be concerned to contrast the firms using only internal finance (retained profits) with those using external sources as well. The 24 firms which used internal finance were asked if they had ever tried to get funds from some other source. *None* of them, it turned out, had ever tried to do so.[2] When asked why, all but three replied that they had no need of external finance, a few adding that they had ' adequate reserves ' or ' plenty of capital '. Two of the remaining three stated that they wished to retain complete control of the business—if they borrowed money ' outsiders would have a say in the business '—and that they wanted ' to keep the business in the family '. The third replied, somewhat cryptically, ' Why expand and get a headache? Success is fictitious.' Thus, the firms which since the war had used only internal finance apparently thought that their profits were adequate for their needs, and had not even considered raising funds from any other source.

Of the 58 firms using external sources, only two stated that they had any difficulty in raising external funds. One gave no details of these difficulties, but it had a large overdraft throughout the period; it had also expanded substantially. The other said that the bank wished to restrict its overdraft, which was also large. This firm was the same size in 1953 as it was in 1947, despite heavy borrowing from the bank and other sources over the whole period. If we may judge from our Birmingham sample, in the post war period the smaller firm has had no difficulty in getting adequate funds to meet its demands. Certainly for our firms, what has been called the ' Macmillan gap ' was virtually closed in this period.

Growth, ' Horizons,' and Profit Levels. The above conclusion, that the finance which firms required was easy to get, raises several interesting questions. Why, in such a period, did many firms rely entirely on internal finance? Was it because they were making very high profits relative to the others, or were their profits adequate mainly because they had relatively much nearer ' horizons '[3] and had set themselves lower targets for expansion?

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[1] Ten firms used one or more of the sources other than overdrafts which are not directly connected with the firm itself, but only 4 firms used these non-bank sources alone.

[2] When some of the respondents said their firms had never tried to do so, they perhaps meant ' never within the memory of the respondent '.

[3] The term ' horizon ' is not used here in the familiar sense of a time horizon (e.g. of a planning period). We mean to convey by it the idea of a level up to which the businessman both wishes to expand and believes it within his power, given time, to do so. The phrase ' target for expansion ', by contrast, conveys only the notion of a particular concrete objective.

Did the firms which resorted to external sources use the money so obtained for expansion, or was the explanation sometimes 'distress borrowing'? We collected a considerable amount of information relevant to these questions, which, taken together, at least suggests something of an answer.

To throw some light on 'horizons' we asked about the firms' growth from 1947 to 1953, about the size that it would like to be in the future, and whether or not it thought it would expand in the next five years. Answers to these questions by no means provide conclusive evidence about what we have called the 'horizon' of the firm, but they can indicate it in some degree. Of course, one would not expect 'horizons' to remain unaffected by the level of profits: on the contrary, it is very probable that firms which have attained a high level of profit will raise their sights, while firms which have made losses, or less profit than they expected, will lower theirs. But even so, one would still expect firms with similar profit levels to vary in their attitude to expansion, and the attitude to be important, given the profit level, in determining whether the firm would be content to rely on its own profits, or would resort to external finance. For this reason we shall first try to establish significant differences between the horizons of internally financed firms and those of externally financed firms. We shall then try to relate these horizons to the profit levels of the firms concerned.

Growth between 1947 *and* 1953. In this section we shall use the proportionate[1] change in the number of employees in the firm between 1947 and 1953 as a gauge of expansion over the period. All other size measures which we have (e.g. turnover or assets) are in value terms, and deflation to take account of changing price levels would involve considerable complications, especially since firms make different products and use different materials. Proportionate change in employment as a measure of expansion also has disadvantages. In particular, in a period which has been characterised by labour shortages one would expect a tendency for firms to expand by substituting capital for labour. For this reason employment changes may rather understate the expansionary tendencies of our firms. Nevertheless we shall use change in employment as the most satisfactory single measure of growth; and it should certainly be adequate for our present purpose of comparing the expansionary tendencies of two broad groups of firms.

TABLE XXVII

Firms Distributed by Annual Average Percentage Change in Employment between 1947 and 1953

	Decrease	*0*	*1–4*	*5–9*	*10 and over*	*Total*
A. Internally financed firms ...	4	9	3	4	0	20
B. Externally financed firms ...	8	11	8	14	13	54

From Table XXVII it is clear that the firms in group A (internally financed) did not expand so much on the whole as those in group B (externally financed). Roughly two-thirds of group A either declined in terms of employment or remained the same size, while only one-third of the externally financed firms did so. In addition, more than a fifth of the externally financed firms grew by more than 10 per cent per annum, while none of the internally financed firms reached this level of increase. There is something of a size effect in this : nine of the eleven firms growing by more than 10 per cent per annum had less than 40 employees in 1953. But there were 5 internally financed firms with less than 40 employees in 1953 and none of them reached this level of increase: the general picture remains substantially unaffected.

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[1] In this section all rates of change will be compound rates.

Such a large difference between the two groups, would seem to suggest that on the whole the externally financed firms, over the given period at least, had somewhat more distant horizons than the internally financed firms.

Future Expectations and Horizons. Is there any connection between past growth and expansion plans for the future? We asked each firm if it intended to expand in the next five years and whether in relation to its industry it regarded its present size as just about right or wished to be bigger. The first question was aimed at establishing the firm's actual expectations in the not too distant future; the second was an attempt to get some idea of what the firm had in mind as a desirable size. Both questions were designed to give some indication of the horizon of the firm.

Only 13 out of the 74 firms answering these questions thought they would not expand in the next 5 years, and these 13 were spread over our whole size range: size and potential expansion were not related. But five of the non-expansionist 13 firms were internally financed and these five form a higher proportion of their group than the remaining 8 externally financed firms form of theirs. Since so many firms expect to expand in the next few years, an attempt to draw further contrasts between the two groups would not be warranted. It is sufficient to state that the short term expectations of the two groups do not contravene our impression that they have different attitudes to expansion. Interesting, however, is the relation between past results and future expectations. Thirty-two of our firms did not expand between 1947 and 1953, but only 13 did not expect to expand in the next five years; future aims were higher than a projection of past trends would indicate.

We may turn now to the question of whether or not our firms are more or less content with their present size, or whether they would like to be bigger. Once again there is a strong suggestion of more distant horizons among our externally financed firms : exactly two-thirds of these would like to be bigger, while only half the internally financed firms are not content with their present size. The firms which have grown in the past are not always those which wish to grow in the future. If among the internally financed group of firms we distinguish two classes, those firms which have expanded previously, and those which have not, then each class splits evenly in half on this question of future expansion. So, too, do those externally financed firms which have not grown in the past. Only those externally financed firms which have grown in the past have a significant majority among them which would like to be larger, more than two-thirds, in fact, as opposed to less than one-third content with their present size. None of this appears to be the effect of size: firms content with their present size and firms wishing to be bigger were both spread through all our size classes. Thus the earlier suggestion that externally financed firms have more distant horizons than internally financed firms is reinforced.

In sum, we have found that firms using external finance over the period appear, in general, to have more distant horizons than internally financed firms. This conclusion derives from three facts, that a much higher proportion of externally financed firms expanded between 1947 and 1953, that a much higher proportion thought they were below optimum size, and that a higher proportion expected to expand in the next five years. That the internally financed firms had no need of external finance, as almost all claimed, might then be explained by their having relatively near horizons, and hence lower expansionary targets. But before accepting this as a hypothesis it is desirable to look at profit levels and relate them to the horizons of the firms.

Profits. As a main measure of profit levels we shall use the ratio of gross profit to total assets. We have already indicated in an earlier section that there are various reasons why the gross profits and the total assets of different firms are not strictly comparable. Nevertheless, while it is as well to be aware of this, too much should not be made of it: certainly for our present purpose of comparing the profitability of two broad groups of firms the rate of gross profit on total assets should be adequate. For good measure, however, we shall later remove from our distribution firms which rent their land and buildings as opposed to owning them: this should at least overcome one major difficulty in the comparison of different firms' assets. We shall also use as a secondary measure the ratio of gross profit to turnover, or the 'profit margin'.

TABLE XXVIII

Firms distributed by Gross Profits as a Percentage of Total Assets and Net Sales (Turnover) for 1952 and 1953 (average)

	Gross Profit as a Percentage of Total Assets					
	Loss	*0–4*	*5–9*	*10–14*	*15 & over*	*Total*
A. Internally financed firms ...	1	0	1	7	5	14
B. Externally financed firms ...	11	7	12	6	6	42
	Gross Profit as a Percentage of Net Sales					
	Loss	*0–4*	*5–9*	*10–14*	*15 & over*	*Total*
A. Internally financed firms ...	1	4	3	4	2	14
B. Externally financed firms ...	11	18	10	2	1	42

The general level of profits would seem to be markedly higher among internally financed firms, whether measured as a rate on assets, or as a margin on turnover. More than a quarter of the externally financed firms made a loss and more than a quarter reached a rate of return on assets of 10 per cent and over. But only one of the internally financed firms made a loss and all but two (85 per cent) reached the level of a profit rate of 10 per cent or over. Turnover is generally larger than assets, so that if we take the ratio of gross profits to turnover the distribution moves, as a whole, into smaller classes.

But almost a half of the internally financed firms have a profit margin of more than 10 per cent, while less than a tenth of ' other sources ' firms are at that level. It would appear that the internally financed firms have at the same time both nearer horizons and higher profit levels than the firms using other sources of finance.

There are only two internally financed firms which rent their property. If we impute a value for their land and buildings this lowers their profit rates (17 per cent and 11 per cent respectively, calculated on assets); but these two firms merely move down one class leaving the distribution of internally financed firms virtually unaltered. On the other hand, there were 13 externally financed firms which rented their property, and only one was in the 15 per cent and over class. The remainder had profit rates of under 10 per cent and if values for land and buildings are imputed they would have lower rates still. Thus to impute a value for land and buildings of the renting firms would overall have a much greater depressing effect on the profit rates of the externally financed firms than it would have on those of the internally financed firms. The hypothesis that internally financed firms have higher profit levels and nearer horizons still stands. But we can subject it to a further test if we relate the horizons of individual firms to their profit levels. When this is done we get rather curious results (Table XXIX).

Taking the internally financed firms first, we find that all 5 of the firms in the 15 per cent and over profit class would like to be bigger, but 6 of the 7 firms in the 10–14 per cent class are content with their present size. Similarly content is the single firm in the 5—9 per cent class, while the one firm making a loss would like to be bigger. Turning now to the externally financed, we find the attitude of the two top profit classes is identical. All but one of those earning more than 15 per cent would like to be bigger, while all but one of those earning between 10 per cent and 15 per cent are content with their present size. In the remaining two profit classes, with profit rates between 0 per cent and 10 per cent, about two-thirds would like to be bigger, while only 2 of the 11 firms making a loss are content with their present size.

TABLE XXIX

Firms distributed by Gross Profits as a percentage of Total Assets and by Attitude to Expansion

	Gross Profit as percentage of Total Assets				
	Loss	*0–4*	*5–9*	*10–14*	*15 and over*
Internally financed firms—					
Firms wanting to be bigger	1	—	—	1	5
Firms content with present size	—	—	1	6	—
Externally financed firms—					
Firms wanting to be bigger	9	4	7	1	3
Firms content with present size	2	1	5	5	1

TABLE XXX

Firms distributed by Gross Profits as a Percentage of Total Assets and by Attitude to Expansion

	Gross Profit as percentage of Total Assets				
	Loss	*0–4*	*5–9*	*10–14*	*15 & over*
Firms wanting to be bigger ...	10	4	7	2	8
Firms content with present size ...	2	1	6	11	1

Lumping the two groups of firms together (Table XXX) brings out quite clearly that in all classes except the two with profit rates between 5 per cent and 15 per cent there is a majority of firms wanting to be bigger, and that the majority is marked in the class earning 15 per cent and more and the class making losses. In the 10–15 per cent class the position is reversed, while numbers are fairly evenly split in the 5–10 per cent class. We are, admittedly, dealing with numbers which are too small to enable us to come to any very definite conclusions. But we have some indication that profit levels do affect the horizon of the firm, and in a peculiar way. The impression we gain is strengthened if we relate profit rates both to past growth and the desire for future expansion : all the firms with a profit rate of 15 per cent or over have either grown or want to grow and three-quarters of them have both grown and still want to grow. *None* of the firms in the 10–15 per cent class has both grown and still wants to grow, and half of the firms in the class have not grown and are content with their present size.

The difference between these two top profit classes is striking : it strongly suggests that a profit rate of about 10 per cent to 15 per cent is a comfortable one, that firms earning this kind of rate think they are doing well enough and bend their efforts to preserving their position without thinking much of expansion—apart, perhaps, from the risks it would entail. Above this level of profits, firms seem to think predominantly of expansion. Below this level the wish to expand is again very much stronger than at this level, but not so strong as above it. About a quarter of the firms with profit levels of 0 per cent to 10 per cent have not grown in the past and are content with their present size, and roughly a half have both grown and want to grow still further. The majority of firms in this group have distant horizons, although the proportion is smaller than in the top profits class.

Among those which made a loss over the last two years, as in the lower profit classes, half had both grown in the past and wished to grow in the future; and there was only one firm out of 12 which had not grown and did not want to grow. It would seem that low profits or losses, particularly perhaps losses, tend to make firms think of expansion as the remedy for their ailments.

From this discussion of the linkage between horizons and profit levels, we can proceed to say something further about our two groups of firms. The internally financed firms lie almost entirely in the classes which have a profit rate of more than 10 per cent. Most of them fall in the 10 per cent to 15 per cent class and this is the group of firms which we implied was the ‘ comfortable ’ group—with enough profit to make the firm happy, but not so much as to make it wish for more. When we said that on the whole the internally financed firms had nearer horizons than the others, we meant that a *higher proportion* of them had near horizons. We now find that the firms which, in the internally financed group, are responsible for this higher proportion of near horizons almost all fall into the profits class 10 per cent–15 per cent. Further, whereas half the internally financed firms are in this class, only one-seventh of the externally financed firms are in it. And while three-quarters of the externally financed firms had profit rates of less than 10 per cent, only 2 of the internally financed firms had rates as low as this. Consequently, it might not be unreasonable to suggest that the bulk of the firms using external finance had distant horizons because they had a low level of profits, that they wished to expand because they wished to improve their profits; but that because their profits were low, to put their wishes into effect they were forced to resort to sources of finance other than profits.[1]

We have, however, not yet examined the two small classes of externally financed firms with profit rates of more than 10 per cent. The top class, with a rate of 15 per cent or over, have the same distant horizons as the corresponding class of *internally* financed firms, and it is perhaps significant that by 1953 no firm in the class had any external finance outstanding on its balance sheets. It would seem that all firms in the class had not only repaid all loans but also intended now to expand out of profits, and had, in fact, already become internally financed firms. The six externally financed firms with profit rates between 10 per cent and 15 per cent had the same near horizons as the internally financed firms in the same class. Two had by 1953 paid off all loans, two more had repaid a considerable part of their earlier commitments, and the remaining two had borrowed heavily and expanded rapidly in the last three years but now thought they were big enough. It is at least possible that, given time, these firms, if their profit remains at the same level, may also become internally financed firms.

In this explanation of why some firms use profits only, while others resort to other sources of finance, the level of profits plays an all-important role. On the one hand the level of profits partly determines the horizon, on which depends the firm's demand for investible funds; on the other hand profits are also the most important source of supply of such funds. Broadly speaking, internally financed firms are either firms with a *very* high rate of profit which have recently expanded and wish to expand further, presumably because of this very high level, or they are firms with a *fairly* high rate of profit which have not expanded in the past and do not wish to do so in the future, presumably because their profits are high enough to be ' comfortable ' but not high enough to push them into taking the risks of expansion. In other words if one is to depend solely on one's own profits, they must be at a comfortable level; and it would seem that if they are just high enough for that (10 per cent –15 percent ?) then one's horizon adapts itself to that level. If they are very high (above 15 per cent ?) then, presumably, the owners can have a reasonable income while the firm can expand *out of profits* at the same time. The essential point would seem to be that most firms would prefer to finance themselves out of their own profits—but, of course, unless the profit level is high it is not possible to do this.

On the other hand, most firms using external finance have a rate of profit lower than 10 per cent, a rate which by and large, it appears, is unsatisfactory; hence the distant horizons and the use of external finance to bring about the expansion which, it is hoped, will raise profits.

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[1] Some of their additional finance over the previous 6 years was, however, not used for expansion. Half of the firms making a loss did not grow in that period, and their additional funds could perhaps be classed as borrowing for the purpose of keeping afloat.

So far we have found that profit rates, in conjunction with horizons, appear to provide some explanation of our firms' financial behaviour. But profit rates are only one side of the story; we need to look at the size of the firm and the absolute amount of profit (not forgetting directors' remuneration), if we are to judge whether or not a high rate of profit provides a comfortable living for the owners of the firm concerned. Clearly profit rates could be high but assets small, so that the amount available as profit would be quite insufficient to provide anything but a poor income for the owners and leave nothing for the firm's expansion.

In Table XXXI directors' remuneration has been added back to gross profits. While, for the reasons mentioned earlier, this does not make the profits of different firms more comparable, it does show the earnings of the firms out of which are provided the directors' wage cost, and directors' surplus remuneration, dividends and the retained profits, which, after due deduction for tax, are available for ploughing back. In other words, it shows the fund from which (if there were not other sources of finance) the various appropriations could be made.

All the internally financed firms except one had more than £1,000 available, and all those with more than 100 employees had over £5,000 (only one of the 5 concerned having less than £20,000). It is to be noted, moreover, that profits plus directors' remuneration increase with size. Although businessmen's opinions about what constitutes a comfortable level of earnings

TABLE XXXI

Distribution of Firms by Gross Profits and Directors' Remuneration, 1952 and 1953 (average), and by Size Classes

Profits and Directors' Remuneration	*Employment*				
	10–19	*20–49*	*50–99*	*100 & over*	*Total*
Internally financed firms—					
£20,000 and over	—	1	—	4	5
£10,000—£19,999	—	—	1	—	1
£5,000—£9,999	—	—	1	1	2
£2,000—£4,999	—	2	1	—	3
£1,000—£1,999	—	2	—	—	2
£0—£999	—	—	—	—	0
Loss	—	1	—	—	1
Total	0	6	3	5	14
Externally financed firms—					
£20,000 and over	—	—	1	3	4
£10,000—£19,999	—	—	2	2	4
£5,000—£9,999	1	1	3	2	7
£2,000—£4,999	2	7	2	—	11
£1,000—£1,999	2	4	1	—	7
£0—£999	4	1	—	—	5
Loss	1	1	2	—	4
Total	10	14	11	7	42

are bound to differ, only one[1] of the internally financed firms could really be said to have earnings that were not at a comfortable level. That is to say, while we cannot establish, with the data available, that the amounts earned were viewed by the firms as a reasonable return on their capital, none of the firms, it appears, has done badly, and most have done well.

With the externally financed firms, as one would expect, there is a similar correlation between size and the amount of profit. Only three, however, of the 7 firms employing a hundred or more men made as much as £20,000; but, like the internally financed firms of similar size, none of the 7 made less than £5,000. Among the smaller size classes there is a very wide scatter, but while only one of the internally financed firms made less than £1,000, a quarter of the externally financed firms that had less than a hundred employees had profits as low as this.[2] [3]

There is, therefore, some reason for believing, as we earlier suggested, that a number of externally financed firms do not feel content with their present level of profits. But from Table XXXI we can now see that if this is true, it is in great part an effect of size differences. None of the internally financed firms had less than 20 employees (this is also true of another ten internally financed firms whose profit rates were not obtained) and a large proportion (more than a third) had over a hundred employees. Moreover, a quarter of the firms using external sources had less than 20 employees and only a sixth of them employed more than 100.

At the same time, from the fact that all the firms with less than 20 employees were in the externally financed group we may draw the not surprising conclusion that before firms can expand comfortably out of profits they have to be at least of a reasonable size. It is in keeping with all this that the internally financed firms tended to be bigger, to have higher profit levels, and more often than the externally financed firms to have near horizons.

This brief examination of the relation between absolute amounts of profit and the size of the firm at least does not invalidate the conclusions of our earlier analysis based on profit rates; but it does suggest that size may be an important factor influencing the type of finance used. So far, however, we have only examined the two broad classifications, ‘ internal ’ and ‘ external ’ finance. Since internal finance is defined as retained profits only, our category of external finance includes many varied sources, at least one of which, directors' loans, may even be ' internal ' in a sense akin to the one in which we are using the term. Moreover, we have said nothing about the various sources of external finance or the amounts involved, or the effect of ownership structure or size on methods of finance, or many similar matters. Much of this will have to wait for a detailed analysis of the structure of assets and liabilities which is likely to be the next stage of our work, and which may well entail a modification, perhaps even a revision, of our provisional findings.

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[1] This firm, which was the only one to make a loss among the internally financed firms, lost more than £1,000 in both 1952 and 1953, and paid no dividends or directors' remuneration in these years. However, with existing assets of £20,000 it intended to spend a further £20,000 in 1954 on new assets and was getting the finance from directors' loans. As it intended to get external finance to expand in the future, it could quite easily be treated as an externally financed firm, when it would no longer stand out as a deviate from our suggested patterns of behaviour.

[2] We have mentioned at several stages that there was no *apparent* size difference between the two groups but this was only in so far as in each group there were firms of all sizes with horizons and profit levels of all kinds.

[3] One interesting effect of adding back directors' remuneration is that 7 of the 11 firms which we previously classed as making a loss are now classed as making a profit. Thus some of the firms which we previously thought of as doing rather badly may on this later classification be considered to be doing quite well (e.g. those of them earning more than £2,000).

Epilogue. Some of the horses in this exercise we have probably flogged too hard, and many are obviously limping. Part of the explanation of this state of affairs is to be sought in the tantalising nature of pilot surveys. Our survey, like others of its kind, whets appetites it cannot satisfy : it throws up many interesting facts but few which can be taken with confidence as widely representative; and it suggests lines of analysis without providing the means to explore them thoroughly. For the rest we must admit that, for lack of resources and time, we have not always been able to wreak the best out of the material at our disposal. The reader accordingly should not concentrate too much on the end products of our survey. He should take this report as an attempt to illustrate the potentialities and limitations of the method in a particular field of business research.

THE SAVINGS SURVEYS AND OFFICIAL SOURCES

A RECONCILIATION OF ESTIMATES OF PERSONAL INCOME[1]

BY K. H. STRAW

The Savings Survey estimates of personal income can be compared with those provided by the Central Statistical Office in the National Income blue book. Two earlier comparisons have been made, the first using the findings of the 1952 Savings Survey[2] and the second those of the 1953 Survey.[3] This article arises out of further work on the reconciliation of the Survey data with the estimates of the Central Statistical Office. As this Office has now kindly provided us with the extra detailed information necessary, it is possible to improve on the two reconciliations published earlier; at the same time enough results of the 1954 Survey are now available for us to make a similar comparison of that year's findings with official figures. For technical reasons it is more convenient to handle the reconciliations for 1953 and 1954 together at the beginning of this article, leaving that for 1952 to be dealt with afterwards. It should be borne in mind that, strictly, the Survey figures relate to the financial years 1951–52, 1952–53 and 1953–54, while the corresponding C.S.O. figures relate to the calendar years 1951, 1952 and 1953.

It should be observed that although we appraise the Savings Surveys in terms of the percentage of adjusted C.S.O. estimates covered this does not mean, of course, that we regard the percentage covered as the sole, or even major, criterion for the accuracy of the size distributions of income. A very high percentage coverage would not in itself indicate in any sense that a survey was free from bias or other inaccuracy.

Differences of a few percentage points should not be taken as of much importance. One survey cannot be regarded as being better than another simply because it accounted for several more percentage points of money income than another : we do not consider the estimates of the money income of consumers presented sufficiently accurate for such a conclusion to be drawn. Further, the field surveys cover only small samples so that the ' blown-up ' aggregates, and hence the percentages of income covered, are subject to sampling variations.

In different years special efforts have been made—by changes in survey techniques and definitions, or by altering or adding to the questions —to obtain improved data for specific items, where there was particular interest in such items at the time or where it was thought that there was doubt about the reliability of earlier surveys. Though such efforts at improvement are highly desirable in the long run, they naturally affect year-to-year comparability.

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[1] I am indebted to the Central Statistical Office for assistance with the reconciliation of the items in the official National Income Estimates with the Savings Surveys data. Mr. J. E. Fothergill of the Social Survey helped to design the questionnaires and to plan the Surveys, and supervised the field work and tabulation of the results, and I am indebted to him for invaluable assistance in numerous ways.

[2] H. F. Lydall, ' National Survey of Personal Incomes and Savings, Part II: The Distribution of Personal Incomes'. BULLETIN, Vol. 15, Nos. 2 and 3, Feb. and March 1953, pp. 82–83.

[3] T. P. Hill, L. R. Klein, and K. H. Straw, ' The Savings Survey 1953: Response Rates and Reliability of Data.' BULLETIN. Vol. 17, No. 1, Feb. 1955, pp. 115–116.

The starting point for the reconciliations with official figures is *Table II—Personal Income and Expenditure*—of the *Preliminary Estimates of National Income and Expenditure* 1948–1954 (*Cmd.* 9423): adjustments have been carried out to these official figures to make them correspond as nearly as possible with the Survey definition of personal income.

It has not previously been considered reasonable to adjust for the main difference in coverage between the Survey and official sources—the exclusion from the Survey of parts of the institutional population (persons in non-private households) and of Northern Ireland. We shall try later in this note to indicate the results of an attempted adjustment for these two sections excluded from the Survey. But first of all we will confine our attention to more firmly based figures.

For the purposes of this present comparison the C.S.O. figures for the calendar years 1952 and 1953 have been adjusted as nearly as is possible on present information to correspond with the Survey definition of personal income in the financial years 1952–53 and 1953–54.[1]

The following are the adjustments made to the C.S.O. data :—

DEDUCTIONS FROM C.S.O. FIGURES

Employment Income

Pay in cash and kind of the armed forces.
Wages and salaries in kind—civilian.
Employers' contributions to National Insurance.
Employers' contributions to occupational pension funds.

Income of Self-employed

Depreciation provisions for professional persons, farmers and other sole traders and partnerships.
Farm income of companies.

Rent, Interest and Dividends Received by Persons

Tax on building society dividends and interest.
Accrued interest on National Savings Certificates.
Property incomes of : Ordinary life funds gross.
Industrial life funds.
Friendly societies.
Collecting societies.
Pension funds.
Non-profit-making bodies.

[1] Some of the money amounts involved in these adjustments are rough estimates only. Among the more uncertain figures are those relating to non-profit-making bodies.

Government Transfers

Grants for universities, sciences and the arts.
Other grants to non-profit-making bodies.
Nutrition and health.
Industrial services for the disabled.
Agriculture, forestry and fishing—grants for research.
Employment—training and resettlement.
Industry and commerce—grants for scientific research.
Post-war credits.
Miscellaneous.

ADDITIONS TO C.S.O. FIGURES

Employment Income

Employees' expenses for tools, clothing, etc., allowed by the Inland Revenue.
Occupational pensions (excluding lump sum and death benefits).

Rent, Interest and Dividends

Annuities received by individuals.
Bank interest paid by consumers.
Mortgage interest to building societies and other lending institutions in the non-personal sector.

In general it will be clear that deductions from or additions to the C.S.O. figures are made simply to organise the two sets of data under the same set of definitions. But it may not always be obvious why particular items are included in the list of adjustments. One type of adjustment is made because ' persons ' are usually defined in the official national income estimates to include with individuals ' collective persons ', such as non-profit organisations providing services to individuals, life funds of assurance companies, private trust funds, and private pension funds. The Savings Surveys cover income units in private households; consequently the Survey estimates exclude, in contrast with official estimates, income accruing to these funds or organisations which was not distributed to income units.

Certain other collective items of income included in the official estimates under the head of Government Transfers also have to be excluded in making the adjustments, since items such as certain grants for scientific research or for nutrition and health are received in the first instance by institutions or are disbursed in non-monetary form to individuals.

Again, the Survey does not regard employers' contributions to National Insurance or to occupational pension funds as part of the income of the income unit; the C.S.O. treats such items as part of the employee's income deducted at source.

Another type of conceptual adjustment is that necessary to allow for interest payments made by persons to ' non-persons '. The C.S.O. figures under the head of Rent, Interest, and Dividends in the personal income tables of the blue books are net figures, consisting of receipts less payments. Both sides of the personal income and expenditure tables are deficient by the amount of interest payments by the personal sector to ‘ non-persons ’ such as banks and building societies, since such payments are regarded as negative income and also are not included in expenditure. Bank interest, for instance, is treated in the national accounts as a transfer payment. Thus to get the figures from the sample Surveys and the official estimates on the same basis it is necessary to add to the C.S.O. estimates of personal income bank interest paid by consumers, but not that paid by the self-employed, since their interest payments are treated as business costs in arriving at business income. The same general point applies to mortgage interest, although this is not treated as a transfer payment in the official figures.

Personal income totals in the blue books do not contain an adjustment for depreciation on trading incomes, although an auxiliary table of statutory amounts is included at the end of the blue books. But the questions put to self-employed respondents in the Savings Surveys of 1953 and 1954 make an explicit request for figures of net profit after depreciation charges have been taken into account. And the request is made implicitly in the questions used in 1952.

There are some companies owning farms and their income is included in the C.S.O.’s estimates of total income of the personal sector. Information about such incomes would not be covered by the Surveys and consequently this item is deducted from the official figures.

The tax on building society dividends and interest is paid by the societies under a special agreement and thus the respondents in the Savings Surveys are not likely to know how much tax has been paid on income from this source. The tax paid is included in the C.S.O. estimates and thus needs to be deducted. Accrued interest on National Savings Certificates is not obtained from respondents in the sample Survey and is thus deducted from the C.S.O. personal income figures. Post-war credits are not included in the Survey definition of income but are treated as a capital transfer, and a further deduction has to be made under this head.

Finally, under items which have to be added to the C.S.O. figures to make the personal income estimates agree with the Survey definition of income, is employees’ expenses for tools, clothing, etc., allowed by the Inland Revenue. In the official estimates of personal income this item is deducted before arriving at the total gross cash earnings of wage earners. The amount of this deduction has to be added back into the C.S.O. data to bring about the necessary correspondence with Survey definitions.

The additions to the C.S.O. personal income figures amounted to £285m. in 1952, and £313m. in 1953; the deductions to £1,508m. and £1,614m. in these two years respectively.

The comparisons of the components of personal income are indicated in Table I. Private transfers are omitted from the comparison since there are no published figures of aggregates. The survey totals of the four components are arrived at by blowing up the sample means on the assumption that there are 22 million income units in private households in Great Britain. This estimate of 22 million has been calculated from the 1951 Census.[1]

A further calculation has been done on the basis of dividing the numbers of persons per income unit which emerged from the Survey results into the total population of Great Britain (excluding the institutional population), and this gives an almost identical result with the direct calculations from the Census. Between June 1953 and June 1954 the population of Great Britain increased by approximately 190,000[2]; and this represents about 88,000 income units. Taking rounding off errors into account we still arrive at a total of approximately 22 million income units for 1954.

It will be observed that the figures from the 1954 Savings Survey compare better with the adjusted C.S.O. data than the 1953 figures, particularly those for the income of the self-employed and for unearned income. Nevertheless the Survey totals for these two components still fall a long way short of the adjusted C.S.O. totals, and the same causes for the relatively poor estimates are probably at work as were commented upon at some length in the case of the 1953 Survey data.[3] The Surveys are still much better at estimating income from employment and government transfers than the incomes of the self-employed and rent, interest and dividends received by persons.

One noteworthy result is the sharp decline in 1954 in the proportion of government transfers (according to C.S.O. adjusted figures) covered by the Survey estimate. Whereas in 1953 the Survey estimate amounted to 99.5 per cent of the adjusted C.S.O. figure, in 1954 the proportion fell to 89.3. There appear to be several possible explanations for this. Firstly, of course, it may arise from sampling errors. Secondly, the 1952 calendar year figures of the C.S.O. contain three months of the increase in old-age pensions introduced in October 1952, while the financial year covered by the 1952–53 Survey contains six months at the higher rates, and this may help to account for the almost exact correspondence of the figures from the two sources in 1953. Thirdly, there was a change in the sampling frame in 1954. The basis used in the two earlier years was the rating lists of the local authorities;

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[1] *Census 1951, Great Britain. One Per Cent Sample Tables.* General Register Office. There is some doubt about the precise number of income units to be estimated for the total population in private households in Great Britain. One can get different estimates according to the assumptions made, in the main about the excess of married women over married men, and the lower and upper estimates differ by a maximum of about 800,000 income units. In using 22 million income units in our calculations we are using the most conservative estimate. A case can be made out for assuming 22.8 million income units, which, with rounding, would give us a blow-up factor of 23 million. If we were in fact to use this latter figure and were to make the further assumption that the additional one million income units comprise average survey income units, then we should increase the aggregate personal income covered by the survey by £424m. in 1952, £434m. in 1953 and £477m. in 1954. This, of course, results in a closer approximation between the survey aggregates and the official estimates.

[2] *Statistical Review of England and Wales for 1953, Part II, Civil.* General Registry Office, 1955. H.M.S.O.

[3] T. P. Hill, L. R. Klein, and K. H. Straw, ' The Savings Survey, 1953 : Response Rates and Reliability of Data.' BULLETIN, Vol. 17, No. 1, February 1955.

TABLE I

Comparison of the Components of Personal Income

Type of Income	*C.S.O. (adjusted) data 1952*		*Survey data 1952–53*		*C.S.O. (adjusted) data 1953*		*Survey data 1953–54*		*Survey data as Percentage of C.S.O. (adjusted) data*	
	£m.	%	£m.	%	£m.	%	£m.	%	*1952–3* %	*1953–4* %
Employment income	8,255	71.4	7,458	78.9	8,727	71.1	7,964	76.7	90.4	91.3
Income of self-employed	1,410	12.2	770	8.1	1,463	11.9	1,012	9.8	54.6	69.2
Rent, interest and dividends received by persons	1,126	9.7	462	4.9	1,227	10.0	638	6.1	41.0	52.0
Government transfers	774	6.7	770	8.1	862	7.0	770	7.4	99.5	89.3
Total	11,565	100.0	9,460	100.0	12,279	100.0	10,384	100.0	81.8	84.6

whereas in 1954 the sample was drawn from the Register of Electors.[1] The names of all persons liable for jury service in England and Wales are marked in the electoral register with the suffix J. Addresses containing the jurors composed the upper stratum of the sample in the 1954 Survey, and addresses containing no jurors formed the lower stratum. This stratification does not correspond with the two strata of the samples in the two earlier years, when the basis was generally that of a rateable value of more or less than £30.[2] One of the main objects of the change in the sampling frame between 1953 and 1954, and of themore intensive sampling of the upper stratum, was to try to obtain a better coverage of the higher income groups.

No question was asked in 1954 about the occupation of the head of the income unit, in the case of failures, because of a re-arrangement in the questionnaire designed to facilitate the interviews with co-operative respondents; thus no estimate of the occupational response rate can be calculated. Nevertheless the proportion of income units in which the head was self-employed increased from 4.4 per cent in 1953 to 7 per cent in 1954, and the proportion of income units in which the head was classified as managerial or technical increased from 7.3 per cent to 9.6 per cent. These 1954 proportions of income units in which the heads were either self-employed or in the managerial category are actually greater than the corresponding estimates derived from the 1951 Census of Population. It would appear probable, therefore, that a comparatively good response was in fact achieved in the 1954 Survey from income units in which the heads were in one of these two categories, which provide a large part of the higher income groups in the community.

Fourthly, the mean figures of government transfers in particular occupational classifications have been affected by changes in the coding of occupations—especially in the 1954 Survey. Thus the increase in the proportion of income units in the 1954 Survey in which the head was classified as managerial may in large part be due to coding; but this is not a feasible explanation for the increase in the self-employed whose numbers coding changes are unlikely to have affected, since self-employment is fairly easy to identify. It would therefore seem likely that the increase in the self-employed is due either to improved response or to sampling error.

The mean value of government transfers was £35 in both the 1953 and 1954 Surveys, but the mean of those actually with income from such transfers fell by £2, from £69 to £67, compared with a rise in the national aggregate shown in the adjusted C.S.O. figures of £88 million. This latter figure reflects the general rise in payments out of the National Insurance Fund, increases in industrial injury benefits, in the payments under the family allowances scheme and so on.[3]

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[1] For the advantages of using the jury qualification as a method of stratification see T. Corlett, 'A use for the jury qualification in sample design '. *Applied Statistics*, 1952, pp. 34–36.

[2] For a more detailed account of the sampling procedures adopted see BULLETIN, Vol. 17, No. 1, February 1955 and Vol. 17, No. 2, May 1955.

[3] *Report of the Ministry of Pensions and National Insurance for the year 1954. Cmd. 9495.* H.M.S.O.

The average amount of government transfers is substantially less in the case of income units in which the head is self-employed or in the managerial group than the average for all occupational clasifications, and thus the improvement in coverage of the self-employed and managerial group would tend in fact to bring down the average amount of government transfers in the 1954 Survey.

Further, compared with 1953, there was a decline in the proportion of income units in which the head's occupation was classified as retired or unoccupied,[1] and in this instance government transfers are high—the mean of those in this category with such income being £117; this would again result in a reduction in the estimate of government transfers. Finally, the proportion of income units of which the head was classified as unskilled manual declined by almost 4 percentage points between 1953 and 1954 ; this may in part be due to coding changes. This group has a high government transfer income—in all years second only to the retired and unoccupied—and any decline in coverage would lead to a further reduction in the estimate.

Apart from the official revisions to *National Income and Expenditure* 1946–1953 (used for the earlier published comparison) given in *Cmd.* 9423 (used as the basis of the present comparison), the main changes in the treatment of components of personal income affecting our estimates and comparisons are four in number—a change in the treatment of the pay in cash and kind of the armed forces; of employers' contributions to occupational pension funds; of farm income of companies; and of bank and mortgage interest paid by individuals. In the case of the pay, both in cash and kind, of the forces, it is considered that we achieve a better comparison between the Survey and C.S.O. figures if we deduct this item entirely from the C.S.O. personal income figures. A thorough review of the estimates of ' other employers' contributions to occupational pension funds, etc.' has been made by the C.S.O. in the light of estimates contained in the *Report on the Economic and Financial Problems of the Provision for Old Age* (*Cmd.* 9333) and other information. This item now represents a much more comprehensive estimate than was previously possible of employers' contributions to superannuation and other pension funds. The farm income of companies has been revised to agree with the latest estimates of net income from farming given in the *Annual Review and Determination of Guarantees*, 1955 (*Cmd.* 9406). Finally, a more refined analysis has been supplied by the C.S.O. concerning bank and mortgage interest paid by individuals, which enables us to make a much more valid comparison between the Survey estimates and the C.S.O. data.

The reconciliation up to this stage can be achieved with a fair degree of confidence. Once we try to calculate the allowances for the difference in coverage between the Survey and the C.S.O. data—the exclusion from the Survey of Northern Ireland and the institutional population—we are engaged in a more dubious operation. However, such an attempt is probably worth-while if only to try to show the right order of magnitude attaching to the problem.

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[1] T. P. Hill, ' Incomes, Saving and Net Worth.' BULLETIN, Vol. 17, No. 2, May 1955, p. 165.

Adjustment for Northern Ireland. The population of Northern Ireland was 1,384,000 in June 1953[1] and 1,387,000 in June 1954.[2]

In an analysis of the social accounts for Northern Ireland, the Republic of Ireland and the United Kingdom for the calendar year 1952, kindly supplied by Professor C. F. Carter and Miss Mary Robson,[3] the mean income per head in Northern Ireland is estimated at £194—about one quarter less than the estimate of £253 for the United Kingdom. Thus we obtain an estimate of some £268 million in 1953 and of £269 million in 1954 for income included in the C.S.O. estimates of personal income but not covered by the Savings Survey.

A more refined adjustment is possible, on the basis of the data supplied by Professor Carter, which probably gives a somewhat better estimate. The 1952 totals of each of the four components of personal income can be obtained by calculating the mean income per head for each component in Northern Ireland and then blowing up the results by the estimated number of persons in June 1953 (Table II). Since we have firm estimates for Northern Ireland for the calendar year 1952 only, in adjusting the C.S.O. figures for the calendar year 1953 to the 1953–54 Survey figures we assume that each of the four components of income changed between 1952 and 1953 in Northern Ireland in the same proportion as did the corresponding component in the United Kingdom as a whole. We then obtain the aggregate for each component in the same way as for 1952, after allowing for the increase in the number of persons between June 1953 and June 1954. The estimates of the amounts to be allowed for Northern Ireland and deducted from the C.S.O. figures for the calendar years 1952 and 1953 come to £262 million and £279 million respectively; the details are shown in columns 2 and 5 of Table II.

Adjustment for the Institutional Population. The 1951 Census[4] indicates a total of 2,404,000 persons in non-private households in that year—the details are shown inTable III. No annual estimates are published of the change in this institutional population, so this figure forms our starting point. Eleven types of non-private households are distinguished. These are given in Table III.

Any address which from its description is recognisable as an institution is excluded when the sample is drawn. Institutions whose description is inadequate to allow an exclusion at the time of drawing the sample are excluded by the interviewer in the field when he finds the address given is an institution, except that in the case of hotels, public houses and commercial boarding houses the proprietor or manager and his family are interviewed

[1] *Registrar-General's Statistical Review of England and Wales for the year 1953. Tables, Part II, Civil.* H.M.S.O., 1955.

[2] *Monthly Digest of Statistics.*

[3] The article by Professor C. F. Carter and Miss Mary Robson from which the information on Northern Ireland has been taken will appear in the *Journal of the Statistical and Social Inquiry Society of Ireland, 1954–5,* to be published early in 1956.

[4] *Census 1951, Great Britain. One per Cent Sample Tables.* General Register Office.

TABLE II

Comparison of the Components of Personal Income
£ million

Type of Income	*1952* C.S.O. (adjusted) data[1] (1)	*1952* Northern Ireland (2)	*1952* C.S.O. data (further adjusted) [Col. 1– Col. 2] (3)	*1953* C.S.O. (adjusted) data[1] (4)	*1953* Northern Ireland (5)	*1953* C.S.O. data (further adjusted) [Col. 4– Col. 5] (6)
Employment income	8,255	151	8,104	8,727	160	8,567
Income of self-employed	1410	64	1346	1463	67	1396
Rent, interest and dividends received by persons	1,126	19	1,107	1,227	21	1,206
Government transfers	774	28	746	862	31	831
Total	11,565	262	11,303	12,279	279	12,000
Allowance for Institutional Population ...			274			329
C.S.O. total of personal income after all adjustments			11,029			11,671

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[1] From Table I.

TABLE III

The Population in Non-Private Households in Great Britain, 1951

	Population (000's)
Defence Establishments, including Naval Ships and Service Hospitals	501.2
Educational Establishments and Children's Homes	122.0
Hotels, Boarding Houses, Permanent Holiday Camps, etc., with 10 or More Rooms	288.7
Hotels, Boarding Houses, Permanent Holiday Camps, etc., with less than 10 Rooms	547.2
Institutions for Mentally Ill or Deficient	221.9
Homes for Permanently Disabled or Aged	67.0
Miscellaneous Communal Establishments	201.5
Places of Detention	26.5
Civilian Ships, Boats and Barges	47.2
Others	4.0
Civilian Hospitals and Nursing Homes	377.1
Total	2,404.3

provided they live on the premises; but no guests or staff in such institutions are interviewed. The result of the change in the sampling frame between the first two Surveys and the 1954 Survey will have resulted in different proportions of institutions being excluded at the sampling stage and in the field, although the total exclusions should be constant throughout the years. In 1952 and 1953 more were probably excluded at the sampling stage and fewer by the interviewers in the field; whereas in 1954, as the sample was drawn from the Electoral Register, it is likely that the interviewers would have had to exclude more in the field than they did in the two previous years.

Practically all the people living in defence establishments are excluded from the Survey. This omits not only members of the armed forces (who are eliminated anyway from the comparison) but also civilians who live in such establishments. The majority of the civilians actually working in such establishments are, of course, covered by the Survey since they live in their own homes ; but civilians who both live and work in such establishments are not covered in the Survey. There are no figures available of the numbers involved, and the best we can do is to guess. It is considered that a figure of 50,000 income units would be reasonable.

The only persons residing at ' Educational Establishments and Children's Homes ' who are classified as in an ordinary household and who are thus covered by the Survey are those who are at Boarding School and who are under 16. Students who live at Universities are not included in the income unit of their parents and are thus excluded from the Survey. Here again we have no accurate figures to work from and we consider a guess of 80,000 income units to be not unreasonable. In this case, however, it seems plausible to regard these income units as single person income units rather than average survey income units when blowing up figures to obtain aggregates.

The group ' Hotels, Boarding Houses, and Permanent Holiday Camps with Ten or More Rooms ' is particularly difficult to deal with for our present purposes. The definition of ' Boarding House ' is different in the Survey from that used in the Census. The Census includes a boarding house as a non-private household if it is named as a boarding house or if the total number of boarders in the house is greater than the number of members of the family of the proprietor, including in his family resident staff. According to the Survey definition, however, a house is a boarding house and is excluded if it houses more than three boarders. A further complication is that temporary visitors in such establishments are picked up in the Survey at their own home addresses. The Census was taken in early April, outside the holiday season, and in consequence it seems likely that the proportion of people registered in such addresses who were permanent residents and staff may have been considerable. We assume that half the population of this group—144,000—were not included in the Survey either because they were excluded at the sampling stage or the interviewing stage, or because they were staff or permanent residents.

A comparison of the age distribution of the population in such institutions —based on data in the 1951 Census—with an age distribution based on the Surveys, indicates a substantially higher proportion in the lowest age group, 18–24, which presumably is explained by the presence of resident domestic servants. It also seems probable that a not inconsiderable part of the permanent population of hotels and boarding houses will consist of retired persons. For our present purposes it is assumed that 100,000 of the income units are single person units, and the other 44,000 comprise normal income units. This gives us a total of some 120,000 income units in such institutions not covered by the Survey.

In the case of 'Hotels, Boarding Houses and Permanent Holiday Camps with less than Ten Rooms' it would seem likely that a large proportion of the persons in this part of the non-private household sector are covered in the sample Survey, since households with up to three boarders are included. There were 547,000 persons recorded in the 1951 Census as belonging to this sector of the non-private households. We assume that two-thirds of them are included in the Survey and we are left with 182,000 income units for which allowance has to be made; it is further assumed that these are single person income units.

In general we can consider that the resident staffs of civilian hospitals and nursing homes are excluded from the Survey, but that the patients in such establishments are included. A patient in a hospital has to have been in hospital for more than six months before he is excluded from his own private household by the Survey procedure. Further, in the Survey information is collected by proxy, and the average length of stay in hospital (16 days) is much less than the interviewing period for the Surveys—six to seven weeks—so that it is likely that most members of income units that have been in hospital during the interview period, even of single person income units, are covered in the Survey. We assume that there are 25,000 single person income units resident in such institutions which were not covered in the sample Surveys.

From Table III it can be seen that these five groups of the institutional population make up about three quarters of the total number of persons in non-private households. On the basis of the various assumptions made, we arrive at a total of 94,000 income units assumed to be of average Survey size and 387,000 single person income units not covered by the sample Survey in the non-private household sector.

There is little in the way of adjustment which can be made for the remaining quarter of the population in the non-private household sector, since people in the remaining six categories of institutions are entirely excluded from the sample. The most reasonable assumption appears to be to treat the people as though they were single person income units. The inhabitants of these institutions would seem to be fairly permanent, and also one can expect them to have much lower incomes than an average Survey income unit. Consequently, to our previous estimate of the number of income units in the institutional population not covered in the Survey we must add a further 568,000 single person income units.

Thus the result of these assumptions and adjustments is to leave us with a figure—probably a minimum—of some 94,000 average Survey income units and 955,000 single person income units which have not been either covered in the Survey or already eliminated from the comparison of Survey estimates with the C.S.O. data. We cannot make a detailed adjustment for these income units by the four components of personal income; the best that can be achieved at the present juncture is to assume that the 94,000 income units of the average Survey type have the same mean income as such income units in the Survey—£431 in 1953 and £475 in 1954—and that the 955,000 single person income units have the same mean income as such units in the Survey —£244 in 1953 and £298 in 1954. These means exclude private transfers. The amounts, therefore, which have to be deducted from the C.S.O. data to allow for that part of the institutional population not covered in the Savings Surveys are £274 million in 1952-3 and £329 million in 1953-4 (Table II).

It may be a legitimate objection that it is unrealistic to assume that most of these income units we estimate to be in the institutional population and not covered by the Survey have mean incomes of the same size as the single person income units in private households. One element, however, must be kept in view—that part of the institutional population living permanently in hotels and boarding houses, for which, as we have seen above, we cannot make any really reliable estimates. It seems probable that such persons' incomes are generally larger than the mean income of single person income units, and possible that they are generally larger than the mean income of all income units in private households.

This treatment of the institutional population seems to be the best that can be achieved at present, but it is clearly not fully satisfactory. It should be observed, however, that even if we increase or decrease the estimated aggregate income of the persons in the non-private household sector by 50 per cent, either by altering the estimated number of income units of each type or by altering the assumed mean income of each type, the resultant change in the proportion of the adjusted C.S.O. total personal income covered by the aggregates based on blown up sample means is altered by only about 1 per cent; and given the nature of the whole set of calculations this cannot be considered of much significance.

Reconciliation of 1952 *Survey with Official Sources.* A comparison between the 1951 C.S.O. data on personal income and the aggregates estimated from the first National Survey of Personal Incomes and Savings was published earlier.[1] Information has now been made available by the C.S.O. for the year 1951 similar to that which has been used earlier in this article for 1952 and 1953, and further refinements and adjustments, including estimates for Northern Ireland and the Institutional Population, have been made. This new reconciliation between the 1952 Survey and the 1951 C.S.O. data is shown in Table IV.

We start, this time, with the C.S.O. 1951 figures published in *Table II* of the *Preliminary Estimates of National Income and Expenditure* 1948–54 (*Cmd.* 9423); these are different from those published in the blue book in August 1952, which were used for the earlier reconciliation. There are several points which should be noted about the treatment of the reconciliation for 1952, mainly relating to changes in definition between the Surveys.[2] The main change affecting the reconciliation between the official figures and the Survey data relates to the imputed rent on owner-occupied property (this was taken to be equal to the net annual value of the house for income tax purposes). In 1952 this item has to be deducted from the official figures of rent, interest, and dividends: it was not included in the Survey of that year, nor is it shown in the list of adjustments on pp. 284-5 of this article. A second item included in the concept of gross income in 1953 and 1954 but not in 1952 is income from boarders. A third change relates to the mortgage interest paid on property other than that in which the owner is living; in 1952 this was treated as a deduction to be made before arriving at income and not as an expense to be met out of income.

In calculating the amounts for the four components of personal income for Northern Ireland to be deducted from the 1951 C.S.O. figures we start from the 1952 estimates provided by Professor Carter and assume that the means of the four components in Northern Ireland were less in 1951 than in 1952 by the same proportions as occur in the national income estimates for the United Kingdom as a whole. We then adopt exactly the same procedures as for the 1953 and 1954 reconciliations.

The adjustment for the Institutional Population is carried out in precisely the same way as that used for the reconciliation of the two later Surveys; we use the mean incomes of the average Survey income unit and of single person income units which emerged from the 1952 Survey—£420 and £232 respec-

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[1] H. F. Lydall, 'The Distribution of Personal Incomes.' BULLETIN, Vol. 15, Nos. 2 and 3, February and March 1953, pp. 82–84.

[2] For a detailed account of changes in Survey definitions and techniques between 1952 and 1953 see BULLETIN, Vol. 17, No. 1, February 1955, pp. 123–26.

Table IV

Comparison of the Components of Personal Income
Savings Survey 1951–52 and C.S.O. 1951 Data

Type of Income	*C.S.O. (adjusted) data*		*Survey data*		*Northern Ireland*	*C.S.O. data (further adjusted) [Col. 1 – Col. 5]*
	1951		*1951–52*		*1951*	
	£m. (1)	% (2)	£m. (3)	% (4)	£m. (5)	£m. (6)
Employment income	7,745	72.0	7,200	76.6	140	7,605
Income of self-employed	1,340	12.4	1,270	13.5	60	1,280
Rent, interest and dividends received by persons	1,012	9.4	380	4.0	19	993
Government transfers	664	6.2	550	5.9	23	641
Total	10,761	100.0	9,400	100.0	242	10,519
Allowance for Institutional Population ...						261
C.S.O. total of personal income after all adjustments						10,258

tively—and blow up by the (same) estimated number of income units in that part of the Institutional Population not covered in the Survey.

When all these various assumptions and adjustments have been made we see that the aggregates calculated from the means of the three Savings Surveys have formed significantly different proportions of the adjusted official figures (Table V). Over the three years the proportion of the C.S.O.'s total personal income estimate covered in the Survey has varied from about 92 per cent in the first Survey in 1952 to 86 per cent in 1953 and 89 per cent in 1954. Among the components of personal income the greatest stability in proportionate coverage has been achieved by income from employment, where the Survey has maintained a figure in all three years of between 92 per cent and 95 per cent of adjusted official estimates. There has been a much wider fluctuation in the other components. The proportionate coverage of government transfers has shown a wider variation than that of employment income. The decline in reported income from government transfers which occurred in the Survey figures between 1953 and 1954 has already been commented upon earlier in this article. The increase in the mean income from government transfers between 1952 and 1953 has been discussed in some detail elsewhere.[1] There is a tendency for more and more income units to receive some kind of social security benefit. On the basis of data from the Surveys, between 1952 and 1953 there

TABLE V

Comparison of the Components of Personal Income[1]

Estimates from Savings Surveys as Proportion of Adjusted C.S.O. Data

	Percentages		
Type of Income	*Estimates from 1952 Survey as proportion of C.S.O. (adjusted) data*[2]	*Estimates from 1953 Survey as proportion of C.S.O. (adjusted) data*[2]	*Estimates from 1954 Survey as proportion of C.S.O (adjusted) data*[2]
Employment income ...	94.7	92.0	93.0
Income of self-employed ...	99.2	57.2	72.5
Rent, interest and dividends received by persons	38.3	41.7	52.9
Government transfers ...	85.8	103.2	92.7
Total (C.S.O. data unadjusted for Institutional Population)	89.4	83.7	86.5
Total (C.S.O. data adjusted for Institutional Population)	91.6	85.8	89.0

[1] As we go to press revised figures of National Income Accounts are about to be released and it is estimated that the revisions will result in changes in the fifth and final lines—the overall percentage comparisons—of between one and two percentage points. The main changes occur in the Rent, Interest and Dividends sector, and it seems that the figures in the third line may be reduced by several percentage points in each year.

[2] After adjustment for Northern Ireland but not, except in bottom line, for Institutional Population.

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[1] T. P. Hill, 'Incomes, Savings and Net Worth.' BULLETIN, Vol. 17, No. 2, May 1955, p. 133.

was a rise of 7 percentage points in the number of income units in the sample receiving some kind of transfer income from the government. Clearly, as the proportion of people reaching retiring age increases the number of income units receiving old age pensions will increase; or again there may be a growth in the numbers of people taking advantage of available grants. The amounts paid out in war pensions and children's allowances increased between 1952 and 1953, and there was a growth in the number of claims for sickness benefit, retirement pensions and widows' benefits.[1] Observed increases may also be due to such factors as the influenza epidemic which occurred in January 1953 or an increase in unemployment resulting in more benefits being paid out. One offsetting factor is that increased rates of pensions were introduced in October 1952, and thus only about three months' increase appears in the C.S.O. figures compared with six months in the Survey estimate. But the increase in the proportion of income units receiving some kind of government transfer income between the 1952 and 1953 Surveys and also the rise in the mean amount of transfers received seem to be too large to be accounted for on such general grounds, and it seems that other factors have been at work.

There were changes in Survey techniques between 1952 and 1953, particularly in the layout of the questionnaire and in the way the questions were asked about government transfer incomes, which are considered likely to have improved the information collected from income units. Further, apart from such changes in Survey techniques, the results are affected by sampling errors; and means for particular occupational classifications, as already mentioned, have been affected by changes in the coding of occupations—especially in the 1954 Survey—which are known to have occurred. The difficulties of defining occupations have been commented upon on several occasions in earlier articles on the Surveys. One particular difficulty is where to draw the boundary between the managerial class and the clerical and sales group. Between 1952 and 1953 the proportion of income units in the clerical and sales category increased by 2 percentage points, the mean income from government transfers of income units in this occupational classification rose from £6 to £14, and the mean for those income units actually with an income from this source increased from £28 to £52. A similar problem arises over skilled and unskilled manual workers; but in this case there have not been such wide fluctuations in the means. Further, the 1953 sample contained a greater proportion of retired and unoccupied, with high government transfer income, and a smaller proportion of self-employed, who have a low income from this source, and this will introduce an upward bias in the estimate.

The percentage coverage of income from self-employment has shown wider fluctuation than that of any of the other components of personal income. As can be seen from Table V, the coverage of the Survey ranges from the high figure of 99 per cent in 1952 to a low of 57 per cent in 1953. One of the reasons for the fall between these two years was a drop of about 50 per cent in the number of farmers in the sample[1]; further the mean income of those with an income from farming fell from about £1,600 in 1952 to about £600 in 1953. It seems very doubtful, however, if the whole of this decline can be explained in terms of sampling errors. It is now considered, after the experience gained in later Surveys, that the form of the questions in the business section of the 1952 schedule was such that business income and savings tended to be over-estimated[2]. Improvements in the 1953 schedule (and also in that used in 1954) designed to achieve more accurate estimates of business income and saving thus had the effect of producing a decline in the Survey estimates of income from self-employment, and so of the proportion borne by these estimates to adjusted C.S.O. figures.

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[1] *Cmd. 9159. Report of the Ministry of Pensions and National Insurance for the year 1953.* H.M.S.O.

The fourth component of personal income—income from rent, interest and dividends—has formed a rising proportion of the C.S.O. adjusted figures over the three Surveys. There has been a substantial increase in the proportion of income units having some unearned income. And there appear to be three reasons for this. Firstly, there was a change in the definition of income between the 1952 and 1953 Surveys whereby the imputed rent of owner-occupied houses was included in the Survey. Secondly, there were changes in the questionnaire designed to obtain better estimates of relatively small amounts of interest received, and greater efforts were made to collect such information from respondents. Thirdly, changes in procedure at the editing stage probably resulted in better estimates being achieved.

As noted, in all the comparisons between the Survey and official sources the Survey figures have been on a financial year basis and the C.S.O. figures on a calendar year basis. An independent set of calculations has been provided by the C.S.O. giving approximate estimates of personal income on a financial year basis. Adjustments were carried out by using selected quarterly or monthly data, or by interpolation.

These C.S.O. figures give total personal income for the financial year 1951–52 as about 2.4 per cent higher than for the calendar year 1951; figures for financial years 1952–53 and 1953–54 are respectively 1.3 per cent and 1.2 per cent greater than those for calendar years 1952 and 1953. Unfortunately no C.S.O. figures on a financial year basis are available for separate components of personal income.

The Survey aggregates blown up from sample means should, strictly, be compared with C.S.O. financial year figures. If this is done, as is possible, for total personal income, the percentages of C.S.O. figures covered by Survey estimates come out at about 2.4 per cent, 1.3 per cent and 1.2 per cent less in the three years than those indicated in the last line of Table V.

Comparison of Findings with U.S. Data. In the United States, the only other country in which Surveys of the type under discussion have been consistently carried out on a large scale, the results which emerge from comparing the findings of the Surveys of Consumer Finances and other field surveys with official national income estimates are similar to those which have emerged here. International comparisons of this nature can be misleading and are often dangerous. Nevertheless in this case we are only concerned with broad impressions and the results obtained in the U.S.A. are so similar to those for the U.K. that they are probably of some interest. The estimates of aggregate income of the Survey of Consumer Finances, based on blown up sample means calculated from field studies, range from 83 per cent of the corresponding official estimate of the National Income Division of the Department of Commerce in 1946 to nearly 94 per cent in 1948. Of nine field surveys carried out in 1941 and between 1944 and 1948, four accounted for 74–79 per cent of consumer money income, the other five for 81–92 per cent.[1]

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[1] BULLETIN, Vol. 17, No. 2, May 1955, pp. 131 and 132.

[2] BULLETIN, Vol. 17, No. 1, Feb. 1955, p. 106. BULLETIN, Vol. 17, No. 2, pp. 177 and 178.

The estimates of total personal income of the Survey of Consumer Finances amounted to 92 per cent and 91 per cent of the corresponding Department of Commerce totals in 1949 and 1950.[2] It is estimated that the income surveys conducted by the Bureau of the Census during the past few years have obtained about 85 per cent of the comparable total income aggregates and about 90 per cent of the comparable wage or salary aggregates prepared by the Office of Business Economics.[3]

At the time of writing no later figures of the components of personal income derived from field studies are available for the United States than those for 1946—the Survey of Consumer Finances does not publish regularly mean income by source—and this year is a rather tricky one since such items as armed forces pay, military allowances and allotments, and veterans payments were on a very large scale. However, we have simply excluded such military type payments from the calculations in the same way as the pay of the armed forces has been excluded from the U.K. national income figures. The data published by Selma Goldsmith[1] have been put broadly on the same basis as figures for the U.K. The comparison of the field survey estimates of the proportionate distribution of the components of personal income in the United States and the official estimates of the National Income Division personal income series is given in Table VI.

The pattern of the distributions in Table VI is familiar—see Tables I, IV and V. It is apparent that in the United States the field surveys obtain a distribution of the components of personal income biased towards employment income, and that they fail to obtain satisfactory estimates of the income of the self-employed; of rent, interest and dividends; and also—unlike the Savings Surveys results so far in the U.K.—of government transfers.

Indeed, although it is unwise to judge on the basis of one year, it is worth

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[1] Selma Goldsmith, 'Appraisal of Basic Data Available for Constructing Income Size Distributions,' in *Studies in Income and Wealth*, Vol. 13, published by the National Bureau of Economic Research.

[2] Irwin Friend, with the assistance of Vito Natrella, *Individuals' Saving : Volume and Composition*. John Wiley and Sons, Inc. New York, 1954.

[3] U.S. Department of Commerce, Bureau of the Census. *Current Population Reports: Consumer Income*. Government Printing Office, Washington, D.C., May 1955.

TABLE VI

Components of Personal Income in the United States in 1946

Type of Income	*Percentages*		
	Estimate from Survey[1]	*Official estimate*[2]	*Estimate from Survey as proportion of official estimate*
Employment income	76.7	66.4	92
Income of self-employed	16.9	21.6	62
Rent, interest and dividends received by persons	4.5	9.7	37
Government transfers	1.9	2.3	63
Total	100.0	100.0	79

[1] Survey carried out by the Bureaus of the Census and Agricultural Economics.

[2] Adjusted estimates of the National Income Division of the Department of Commerce.

noting that in 1946 in the United States the survey figures for rent, interest and dividends and for government transfers form a lower proportion of the official estimates than those estimated from the three surveys so far completed in Great Britain. In general, it would seem that associated with an understatement of total income common to the surveys in the United States and in Great Britain there is a marked tendency to understate seriously certain types of income, in particular those received by individuals with a higher than average propensity to save.

In conclusion, it should again be emphasised that the Savings Surveys are not carried out with the main intention of blowing up sample means to obtain national aggregates. Rather the intention is that by taking repeated Surveys the distributions of certain economic variables and the inter-relationships between them can be more firmly established.

SOME CALCULATIONS ON LEAST-COST DIETS USING THE SIMPLEX METHOD

By PETER NEWMAN

This paper presents the results of some calculations on the least-cost diet problem, based on the recently invented group of techniques known as linear programming. Diets for three types of farm animals are presented, and the mixtures of basic feeding stuffs meeting these requirements at minimum cost are calculated. The effects of variations in the prices of feeding stuffs, and of changes in the number of feeding stuffs available are also studied. Certain limitations of the approach are discussed.[1]

INTRODUCTION

The problem dealt with in this paper is interesting and has considerable practical importance. Prior to the invention of linear programming, it could only be solved by essentially unsystematic hit-or-miss methods. With problems involving more than a few variables such tactics prove extremely laborious, and often practically impossible. We now have available, however, a systematic trial-and-error computing technique that enables exact answers to be given. The computing time is not excessive; an experienced computer can easily perform the full calculations of this model in a day, while the effect of variations in prices and in available feeding stuffs can be handled much more quickly.

The rest of this paper is divided into five sections. In the first, I discuss the general problem of least-cost diets. In order to make the treatment self-contained, a brief exposition of the simplex method of linear programming is given in the second part. This section may be skipped without loss by those readers less interested in technical details. In the third part, the data of the specific diets are set out, while the results of the computations are presented and analysed in part four, and certain general problems connected with linear programming are discussed in the fifth and last section.

PART I. THE PROBLEM

The general problem may be posed quite briefly. It is assumed that in order to perform certain specified functions (giving two gallons of milk of a specified fat content per day, laying one egg of specified quality every other day) an animal requires a daily intake of certain amounts of essential dietary constituents such as protein, fibre, phosphorus, etc. These diets are obtained from experiments carried out by dietary experts and are taken as given for the purposes of the present study.

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[1] I am indebted to Mr. K. E. Hunt for advice on many of the agricultural problems involved, and to Miss Jean Morris, who performed all the considerable computations with great skill and efficiency.

These dietary constitutents are present in known proportions in all basic feeding-stuffs, and the appropriate tables are accessible in text books on animal husbandry. It is assumed that each feeding-stuff is freely available at its market-price, and the problem is to select that mixture of feeding-stuffs which will satisfy or exceed all the given dietary requirements at least cost. There are an infinite number of mixtures that will do the job, and the problem is to select the cheapest. This is a formidable mathematical problem if the number of dietary requirements and feeding stuffs is at all large, and it is only recently that techniques have been developed to solve it.

Some modifications to the problem as it stands should now be made. First, I have assumed that an excess above the given dietary level of some or all constituents is not harmful to the animal concerned. This assumption is probably correct for the amounts of excess involved in the present investigation. A more exact dietary specification would give upper as well as lower limits for the various constituents.[1]

Secondly, the average farmer, at least in Britain, does not rely only on purchased feeding-stuffs, but has available home-grown feeds, such as pasture, roots and hay, all of which contain the essential dietary constituents. In a sense, his problem is more difficult, for he has to blend bought feeding-stuffs with his own produce to arrive at an optimal diet. Ideally, he should cost his own produce at some appropriate price, but this is seldom done. It is probably better, therefore, to approach the problem from the viewpoint of a manufacturer of blended feeding-stuffs. I shall assume that he wishes to market a compounded feeding-stuff that will satisfy or exceed certain dietary requirements. He is naturally and properly interested in doing this in the most efficient manner, *i.e.* at least cost. It will further be assumed that the compound is marketed in 100 lb. sacks, and that he therefore wishes to produce the cheapest possible mixture weighing 100 lb. and meeting the specifications. The use of 100 lb. is merely for ease of computation; problems involving alternative weights, *e.g.* 112 lb., could easily be computed.

A final qualification belongs as much in the section on results as it does here. Although not conversant with agricultural methods, nor even with agricultural economics, I have made considerable efforts to obtain realistic data. Nevertheless, some readers, especially those with agricultural experience, may feel that some of the results are unacceptable on *a priori* grounds. This divergence can be explained in at least three ways. First, and probably of least importance, is the possibility that these readers may be wrong; that this exact analysis shows up errors that have long persisted in the industry. For example, one of the similar investigations carried out in the United States[2] indicated that oilcakes were considerably over-priced. A second possibility is that the data are themselves in error, so that any conclusions resulting from operations on them are also in error. A final reason for this divergence might be errors or omissions in the dietary specifications. It might be, for example, that the combination of foods in an optimum diet is altogether unpalatable,[1] *e.g.* in a similar problem for human beings, the optimum mixture might consist of pickled onions, yoghourt, and chocolate nut sundaes with whipped cream and hot fudge sauce—perhaps enjoyable for a change, but not as a steady diet. This points to an incomplete specification, since one should know what diets are excluded on such grounds. In this study we deal with this problem partially by completely excluding these foods which *in toto* are distasteful or harmful to a particular animal *e.g.* fish meal when fed to cows has bad effects on the quality of the milk. The problem of unpalatable *proportions* remains.

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[1] A similar problem was of crucial importance in the linear programming analysis of the blending of certain aviation petrols. See A. Charnes, W. W. Cooper and B. Mellon : 'Blending Aviation Gasolines—A Study in Programming Interdependent Activities in an Integrated Oil Company' : *Econometrica*, Vol. 20, No. 2, April 1952, pp. 135–59.

[2] F. V. Waugh : 'The Minimum Cost Dairy Feed' : *Journal of Farm Economics*, Vol. 33, No. 3, August 1951, pp. 299–310; and W. D. Fisher and L. W. Schreiben : 'Linear Programming Applied to Feed-Mixing under Different Price Conditions', *Journal of Farm Economics*, Vol. 35, No. 4, November 1953, pp. 471–83.

One result, obtainable directly from the use of the simplex method, may be stated here in anticipation of the detailed results given in Part IV. A mathematically trivial (but not intuitively obvious) result of casting the problem into linear programming form[2] is that the cheapest diet will contain no more basic feeding stuffs than there are constraints on the solution; it may contain less. Thus in the first diet given in Part III, where the only limitations are on starch equivalent, protein equivalent, and total weight of the mixture, the optimum mixture will contain not more than three feeding stuffs. In the poultry problem, where there are two additional constraints, the solution satisfying the conditions laid down will consist of not more than five feeding stuffs. The next Part deals briefly with the mathematical and more explicitly with the computational aspects of the simplex method, and may be omitted without serious loss of continuity.

PART II. THE METHOD

In this Part, a brief resumé of the simplex method, which is due to Dantzig[3] is given, stress being laid on its computational aspects.[4]

The problem may be stated concisely thus :

Let $x_1, x_2, \ldots, x_n$ be a finite set of variables, and let $p_1, p_2, \ldots p_n$ be a corresponding set of non-negative numbers (prices). Most economic problems are such that solutions containing negative numbers are nonsensical. Therefore we impose the restrictions

$$x_1 \geqslant 0, x_2 \geqslant 0, \ldots\ldots x_n \geqslant 0 \qquad (1)$$

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[1] I owe this point to Dr S. Vajda.

[2] See G. B. Dantzig : 'Maximisation of a Linear Function of Variables subject to Linear Inequalities,' which is Chapter XXI of *Activity Analysis of Production and Allocation*, edited by T. C. Koopmans. Cowles Commission Monograph No. 13, Wiley, 1951.

[3] *Loc. cit.*

[4] Readers wanting a fuller account are referred to Koopmans, *op. cit.*, Chapters XXI and XXII ; to A. Charnes : 'Optimality and Degeneracy in Linear Programming,' *Econometrica*, Vol. 20, No. 2, April 1952, pp. 160–70 ; and to A. Charnes, W. W. Cooper and A. M. Henderson : *An Introduction to Linear Programming*, Wiley, New York, 1953. A forthcoming book by Dr. S. Vajda may also profitably be consulted.

In addition to these inequalities, certain other inequalities are imposed. Thus the lower limit for the starch equivalent in diet (a) given in Part III may be expressed as follows :

$$a_{11}\,x_{1,} + a_{12}\,x_2 + \ldots\ldots\, a_{1\,19}\,x_{19} \geqslant 71.4$$

where the a_{1j} denotes the proportions of starch equivalent in feeding stuff j, *e.g.* putting x_1 = linseed, a_{11} becomes .740. The weight limitation may be written as

$$x_1 + x_2 + \ldots\ldots + x_{19} = 100$$

The problem of linear programming, then is to minimise (or maximise) the linear functional

$$z = p_1\,x_1 + p_2\,x_2 + \ldots\ldots + p_n\,x_n \qquad (2)$$

subject to the m linear inequalities

$$\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots + a_{1n}x_n \geqslant b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots + a_{2n}x_n \geqslant b_2 \\ \cdot \qquad\qquad\qquad\qquad \cdot \quad \cdot \\ \cdot \qquad\qquad\qquad\qquad \cdot \quad \cdot \quad \cdot \\ \cdot \qquad\qquad\qquad\qquad \cdot \quad \cdot \quad \cdot \\ \cdot \qquad\qquad\qquad\qquad \cdot \quad \cdot \quad \cdot \\ a_{m1}\,x_1 + a_{m2}x_2 + \ldots\ldots\ldots + a_{mn}\,x_n \geqslant b_m \end{array} \qquad (3)$$

and the restrictions (1), repeated below

$$x_1 \geqslant 0,\ x_2 \geqslant 0,\ \ldots\ldots\ldots\ x_n \geqslant 0 \qquad (1)$$

The problem is now stated; the difficulty is to solve it. First, in order to get some idea of the geometry of the problem, let us consider a very simple example[1] illustrated in Figures 1 and 2. Suppose that there are only two variables x_1, and x_2, and only two constraints, represented by the inequalities

$$\begin{array}{l} a_{11}\,x_1 + a_{12}\,x_2 \geqslant b_1 \\ a_{21}\,x_1 + a_{22}\,x_2 \geqslant b_2 \end{array} \qquad (4)$$

These are drawn in Figure (*a*) and (*b*) respectively.

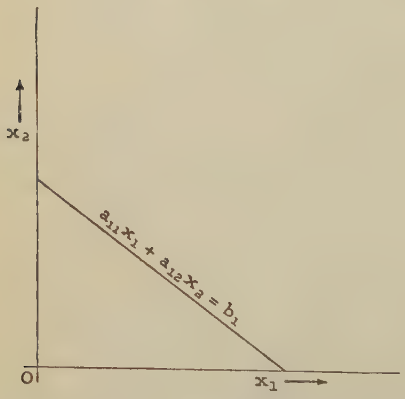

Figure 1a.

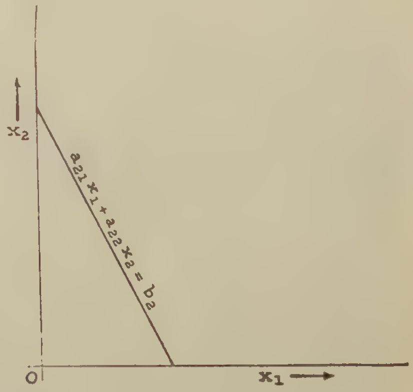

Figure 1b.

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[1] Simple examples along these lines are discussed more fully in Chapter 14 of J. C. C. McKinsey : *Introduction to the Theory of Games*, published for the RAND Corporation by

[Footnote continued on following page

All points lying along the line given by the equation

$$a_{11}\,x_1 + a_{12}\,x_2 = b_1$$

are such that the first constraint is just satisfied; similarly for the second constraint. All points to the right of the line more than satisfy the particular limitation.

Now let us superimpose the two constraints on the same diagram, as in Figure 2. It is convenient to treat two cases, one derived from Figure 1, and another invented for the purpose. In the first case, represented in Figure 2a, the superposition of the two constraints divides the quadrant of non-negative x_1 and x_2 into four disjoint regions, labelled (a), (b), (c) and (d). None of the points in (*a*) satisfy either of the two constraints; those in (*b*) satisfy the first constraint but not the second, while those in (*c*) satisfy constraint 2 but not constraint 1. Only those points in (*d*) satisfy, or more than satisfy, both limitations taken together. The problem, it will be remembered, is to find that combination of x_1 and x_2 which satisfy the limitations at least cost. It is intuitively clear that this optimum combination will occur on the boundary of region (*d*).

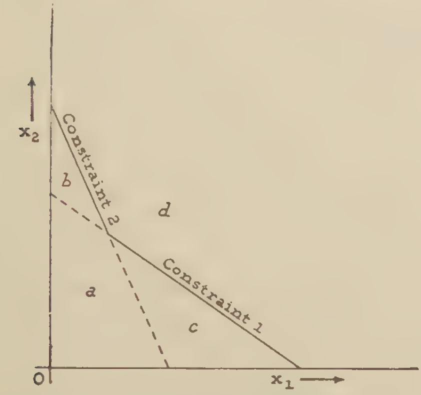

Figure 2a.

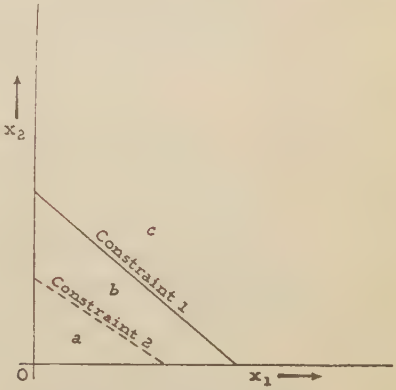

Figure 2b.

Figure 2b illustrates the case where one of the constraints is the dominant one, completely determining what the optimum combination will be. The positive quadrant is now split into three regions; one where neither constraint is fulfilled, one where the second but not the first is fulfilled, and the last where both constraints are satisfied. Constraint 2 is ineffective, for if constraint 1 is satisfied, this automatically guarantees the fulfillment of constraint 2. We shall ignore such constraints in the ensuing development of the theory, though they are common enough in practice.

McGraw Hill, New York, 1952; and in G. Morton: 'Notes on Linear Programming' *Economica*, N.S. Vol. XVIII, November 1951, pp. 397–411. The best discussions of the relation of this approach to the usual techniques of economic theory are contained in R. Dorfman: *Application of Linear programming to the Theory of the Firm*, University of California, 1951; and Dorfman: 'Mathematical, or "Linear" Programming: A Non-mathematical Exposition,' *American Economic Review*, Vol. XLIII, No. 5, December 1953, pp. 797–825.

We shall now demonstrate the solution to the least-cost problem in this two-commodity, two-constraint case. The total cost of the two commodities is given by z, where

$$z = p_1 x_1 + p_2 x_2 \qquad (5)$$

and p_1, p_2 are the prices of x_1 and x_2 respectively. This cost function can itself be represented as a straight line in a diagram similar to Figure 2*a*, with slope $-\frac{p_1}{p_2}$. The problem is to minimise z. Now we may imagine a line with slope $-\frac{p_1}{p_2}$ moving outwards from the origin until it first touches the boundary of (*d*), the area which satisfies both inequalities of (4).

This process is illustrated in Figure 3.

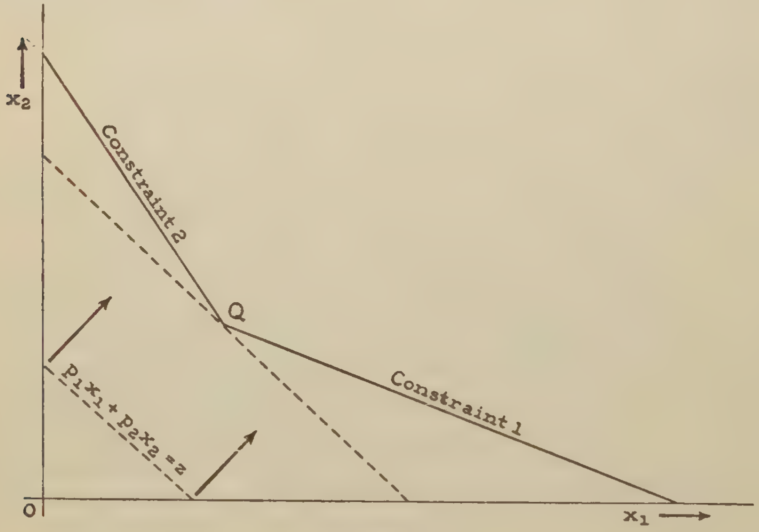

Figure 3.

If the kink at Q is well marked, it is highly probable that the solution is provided by the combination represented by Q. If the price line has a slope less than the slope of constraint 1, however, the solution is found by using x_1 exclusively; and correspondingly for constraint 2 and x_2. There are two borderline cases where the relative prices coincide with either the slope of the first constraint, or the slope of the second. In these cases, equilibrium is indeterminate, the equilibrium combination being any point on the appropriate part of the boundary.

It will be noticed that in the common case, where equilibrium is at Q, quite wide swings in relative prices will fail to result in a different optimum combination. This is a phenomenon that will occur frequently in the empirical results.

More and more constraints may be introduced into the two-commodity problem and the broken line representing the boundary will be broken in more and more places, though remaining convex to the origin. Figure 4 illustrates the case of four constraints.

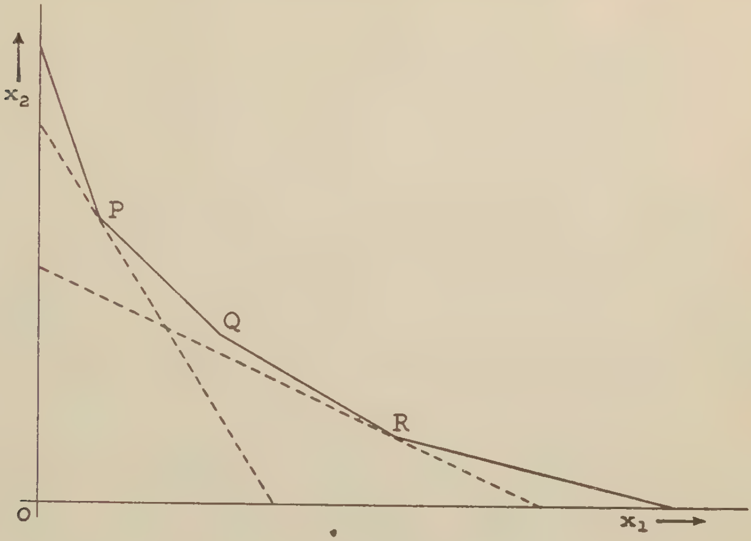

Figure 4.

There is now more likelihood that a fairly large change in relative prices (illustrated by the two dotted lines) will result in a change in the optimum combination, and the likelihood grows with an increase in the number of constraints. Quite wide changes in relative prices may still produce no result, however.

This is in contrast to the treatment of an analogous problem in current economic theory. The problem is one in demand theory: given that a consumer wishes to attain a given preference level, what is the cheapest way of doing it? This is very nearly completely equivalent to the more usual problem of maximising the level of satisfaction attainable with a given income.[1] The solution to the former problem is simply to find the point on the appropriate indifference curve at which the price line is tangential. The trouble with this solution is that one seldom knows the indifference curve; in addition, one postulates a smoothly continuous curve, so that adjustments are made smoothly and not in a series of discrete jumps, as in the linear programming problem. We shall not pursue this analogy here; similar comparisons of the two approaches are more fully discussed in a paper by Dorfman.[2]

[1] For all practical purposes the equivalence is complete. However, delicate cases do exist where the two problems are not isomorphic. See K. J. Arrow and G. Debreu: 'Existence of an Equilibrium for a Competitive Economy'. *Econometrica*, Vol. 22, No. 3, July 1954, pp. 285–6.

[2] 'Mathematical, or 'Linear' Programming: A Non-mathematical Exposition': *American Economic Review*, Vol. XLIII, No. 5, December 1953, pp. 797–825.

It is now perhaps easier to visualise the situation when there are n commodities. The boundary of the set of x's which satisfy the constraints will now look like the surface of an n-dimensional crystal, with innumerable facets. The linear functional which we are trying to minimize will now be an n-dimensional hyperplane which can be imagined to be advancing steadily from the origin of the n-dimensional space towards the crystal, which is suspended in the positive octant of the space. The problem is to determine the point or points (in n-space) at which the hyperplane will hit the crystal. The way in which we solve it is to land on the crystal and crawl over it in some systematic way, from extreme point to extreme point, until we find one or more which we know to be correct. This is not quite the whole story, of course, for one may quite easily pose problems to which no solution exists. However, the crude geometrical picture that has been drawn contains the gist of the matter. The simplex method provides a systematic way of crawling over the surface of the crystal. Further, it has several important properties. If one were to explore the crystal in an unsystematic way there is no guarantee that one would not double back on one's tracks and so on *ad infinitum.* It would be nice to know, also, that at each step things got better and better, bringing one nearer the goal, or at least no further away from it. The simplex method does just this. At each stage one approaches nearer the solution, and at each stage, by a suitable check, one can tell (*a*) whether a solution exists, (*b*) whether one has attained the solution and (*c*) whether further computations are required. We now proceed to a description of the computational details.

The first step in the simplex method is to convert the inequalities (3) into equalities. This is achieved by bringing in so-called 'pseudovariables',[1] one for each inequality, which 'take up the slack' of the inequalities. Thus, if the lower limits are exactly met, the pseudovariable takes the value 0. In this way the set (3) becomes:

$$\begin{array}{l}
a_{11} x_1 + a_{12} x_2 + \ldots + a_{1n} x_n + x_{n+1} + 0 + \ldots + 0 = b_1 \\
a_{21} x_1 + a_{22} x_2 + \ldots + a_{2n} x_n + 0 + x_{n+2} + \ldots + 0 = b_2 \\
\cdot \qquad\qquad\qquad\qquad\qquad\qquad\qquad \cdot \qquad \cdot \\
\cdot \qquad\qquad\qquad\qquad\qquad\qquad\qquad \cdot \qquad \cdot \\
\cdot \qquad\qquad\qquad\qquad\qquad\qquad\qquad \cdot \qquad \cdot \\
a_{m1} x_1 + a_{m2} x_2 + \ldots + a_{mn} x_n + 0 + 0 + \ldots + x_{n+m} = b_m
\end{array} \tag{5}$$

The pseudo-variables have a zero price, so that their presence in the final solution will add nothing to its cost. One difficulty that occurs in our problem is that one of the limitations, that on weight, is a strict equality. In order to achieve this, we adopt the following device (remembering, as Richard Bellman has pointed out, that a device is a trick that works at least twice). To the pseudo-variable occurring in the equation for the weight limitation, we attach a large positive price M, where M need not be explicit, but is taken as many times larger than any other price. No solution which contains this enormous price M will be a minimizing solution, so the value of the corresponding pseudo-variable will be zero. This procedure[1] works for any number of equalities that we may wish to make out of the inequalities (3).

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[1] See A. Charnes: 'Optimality and Degeneracy in Linear Programming,' *Econometrica,* Vol. 20, No. 2, April 1952, pp. 160–70.

It is convenient to regard the set of numbers $a_{11}, a_{21}, a_{31}, \ldots a_{ml}$ as a single point A_1 of m-dimensional Euclidean space, and the column $b_1, b_2, \ldots b_m$ as another point, say A_0, in the same space. The linear programming problem then becomes :

Find $x_1, x_2, \ldots x_n, x_{n+1}, \ldots x_{n+m}$
such that

$$x_1 A_1 + x_2 A_2 + \ldots + x_n A_n + \ldots + x_{u+m} A_{n+m} = A_0 \qquad (6)$$

and

$$x_1 p_1 + x_2 p_2 + \ldots + x_n p_n = z = \text{minimum} \qquad (7)$$

(The problem of maximising z may be treated symmetrically by minimizing $-z$).

The next step is to guess at a solution, that is, to select a set of m A_i's which, multiplied by positive x's will satisfy (6). More succinctly, choose m A_i's such that

$$\bar{x}_1 A_1 + \bar{x}_2 A_2 + \ldots + \bar{x}_m A_m = A_0 \text{ with } \bar{x}_i > 0. \qquad (8)$$

Without loss of generality, we may take the first m of the A_i's to be such a *basis* as it is called. This basis consists, in our problem, of a set of feeding stuffs which can be combined in at least one way to satisfy the dietary restrictions. They need not, and normally will not, constitute the cheapest diet, but they do provide what is known as a *feasible solution.* In small problems such as ours a feasible solution can usually be arrived at very easily, and an inspired guess, which may very well come from those familiar with a given economic problem, may do wonders in shortening the computing time involved. In larger problems a feasible solution is usually not so obvious. However, methods exist for computing a feasible solution,[2] and recent modifications of the simplex method[3] do not in fact require one to start from a feasible solution.

We now try to proceed from a basic feasible solution to an optimum basic feasible solution. To do this, we first express all other vectors $A_{m+1}, A_{m+2}, \ldots A_{m+n}, A_0$ in terms of the basis vectors $A_1, A_2, \ldots A_m$, *e.g.*

$$x_{1j} A_1 + x_{2j} A_2 + \ldots + x_{mj} A = A_j \qquad (9)$$

There are two important special cases of (9). One is simply

$$1 A_1 + 0 A_2 + \ldots + 0 A_m = p_1 \qquad (10)$$

and so on for all A_j in the basis, and the other is

$$x_{1,0} A_1 + x_{2,0} A_2 + \ldots + x_{m,0} A_m = A_0 \qquad (11)$$

i.e. the requirements are expressed in terms of the basis vectors. These

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[1] See A. Charnes, W. W. Cooper and A. M. Henderson : *An Introduction to Linear Programming,* Wiley, New York, 1953.

[2] Koopmans (ed.) *op. cit.*, Chapter 21.

[3] Charnes, *op. cit.* ; G. B. Dantzig and Others : 'Notes on Linear Programming,' Rand Reports, P- 392, 393, 394, 440, 460, 472, October 1953—January 1954 ; E. M. L. Beale : 'An Alternative Method of Linear Programming,' *Proceedings of the Cambridge Philosophical Society*, Vol. 50, October 1954, pp. 513–523 ; C. E. Lemke : 'The Dual Method of Linear Programming,' Technical Report No. 29 of the Department of Mathematics, Carnegie Institute of Technology, Pittsburgh, Pennsylvania, March 1953.

$x_{1,0}, x_{2,0}, \ldots x_{m,0}$ correspond to the $\bar{x}_1, \bar{x}_2, \ldots \bar{x}_m$ of (8), and constitute the levels of the variables whose vectors constitute the basis.

The calculations are best set out in the form of a 'simplex tableau'.[1]

Prices of basis elements	Basis *elements*	*Column vectors*									
		A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
p_1	A_1	$\bar{x}_1$	1	0	0	0	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}
p_2	A_2	$\bar{x}_2$	0	1	0	0	x_{25}	x_{26}	x_{27}	x_{28}	x_{29}
p_3	A_3	$\bar{x}_3$	0	0	1	0	x_{35}	x_{36}	x_{37}	x_{38}	x_{39}
p_4	A_4	$\bar{x}_4$	0	0	0	1	x_{45}	x_{46}	x_{47}	x_{48}	x_{49}
		z_0	$z_1 - p_1$	$z_2 - p_2$	$z_3 - p_3$	$z_4 - p_4$	$z_5 - p_5$	$z_6 - p_6$	$z_7 - p_7$	$z_8 - p_8$	$z_9 - p_9$

We set this out for $m = 4$ and $n = 5$. It should be noticed that there is no reason why the original basis should not contain some pseudo-variables. In fact it often simplifies the task of obtaining an initial feasible solution. Having set out all the vectors in terms of linear combinations of the basic vectors,[2] we then compute certain sums. The initial value z_0 of the linear functional z is given by

$$p_1 \bar{x}_1 + p_2 \bar{x}_2 + p_3 \bar{x}_3 + p_4 \bar{x}_4 = z_0 \qquad (12)$$

The value of, say, z_5 is given by

$$p_1 x_{15} + p_2 x_{25} + p_3 x_{35} + p_4 x_{45} = z_5 \qquad (13)$$

For A_1, A_2, A_3 and A_4 the corresponding z's are simply the prices p_1, p_2, p_3, and p_4, and their entries on the lowest line are zero. For the others we compute $z_j - p_j$ and note whether the result is positive or negative. If all the $z_j - p_j$ are negative then the basic theorems of the simplex method tell us that the minimum has been attained and that z_0 is it.[3] Another case occurs if all the x_{ij} are negative: in this case no minimum exists. The common case, however, is that some of the $z_j - p_j$ will be positive. Then the simplex method tells us that one can lower z_0 further by removing one of the basis vectors and substituting another vector not already in the basis.

We now need criteria to tell us which vector to throw out of the basis, and which to put in. Any vector giving a positive $z_j - p_j$ is eligible to be put into the basis, but empirically it appears that the best results are obtained when that vector, say A_6, giving the largest value of $z_j - p_j$ is put in. In order to determine the vector to throw out, we consider the column vector A_6, compare the ratios $\bar{x}_1/x_{16}$, $\bar{x}_2/x_{26}$, $\bar{x}_3/x_{36}$ and $\bar{x}_4/x_{36}$, and select that basis vector, say A_4, which gives the largest *algebraic* value of the ratio. A_3 is then thrown out, and all the column vectors are recomputed using the new basis A_1, A_2, A_4 and A_6. It is a theorem of Dantzig's[4] that this new basis must result in a decrease of z_0, and so is a step nearer the optimal mixture.

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[1] Charnes, *op. cit.*, p. 165.

[2] As in Charnes, *op. cit.*

[3] Appropriate adjustments have to be made here and throughout the analysis if one is *maximizing* a linear functional rather than minimizing it. See Koopmans (ed.), *op. cit.* Chapter XXI.

[4] Koopmans (ed.) *op. cit.* Chapter 21.

Fortunately we do not have to repeat the (often considerable) work entailed in expressing all the A_i in terms of the original basis. All we need do is to compute the elements x'_{ij} of the new tableau, which are given by

$$\left.\begin{aligned} x'_{ij} &= x_{ij} - \frac{x_{3j}}{x_{36}}\, x_{i6} \\ \text{and } x'_{6j} &= \frac{x_{3j}}{x_{36}} \end{aligned}\right\} \qquad (14)$$

and these are easy to compute.

At this point a phenomenon called ' degeneracy ' may appear, although it did not do so in any of our empirical problems. This is the appearance, in the A_0 column of the new basis, of zeros or negative numbers. This would imply that the solution contains less than m variables *or* pseudo-variables, and is caused by the rank of the $m \times (m + n + 1)$ matrix formed by adjoining A_0 and all the vectors A_i being less than m. Since it did not occur in our problem, we shall not discuss it further in this outline. Recent developments to which the reader is referred[1] have either evolved methods of dealing with it, or involve methods that automatically exclude it.

Exactly the same calculations are performed with the new basis as with the old. If any of the $z_j - p_j$ are positive, a new basis is obtained in a similar fashion, and a new tableau computed. At each stage the value of z_0 decreases and it only ceases to fall when all the $z_j - p_j$ are negative. At this point the cheapest combination has been obtained, and is given by the entries in the A_0 column. Dantzig has observed that the number of stages required is usually of the order of the number of constraints, an observation which is borne out by our experience.

A change in prices requires fresh computation of everything except the original basis, while the deletion of one or more variables may easily be handled by deleting each one from the iteration at the point where it entered the basis, and proceeding from the earliest deletion.

This concludes our survey of the computational technique. Many points e.g. the problem of ties, have not been treated, but interested readers are invited to look up the references cited. The next section gives the data on which the computations were performed.

PART III. THE DATA

In this section the data of the problem are presented. We require three distinct pieces of information (i) the list of feeding-stuffs from which the diet is to be selected, together with the list of appropriate prices (ii) the proportions of the various relevant dietary constituents in these feeding-stuffs and (iii) the specification of the dietary requirements for the assigned task of each farm animal considered.

The feeding-stuffs available for this study were determined by the availability of their prices. These were taken from the Feeding-Stuffs Prices

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[1] See footnote 3, p. 311 above.

Enquiry, a sample survey conducted by the Ministry of Agriculture, who circulate the results each month in mimeographed form. Besides feeding-stuffs already compounded, which do not immediately concern us, these surveys give the average price per ton of twenty uncompounded feeding-stuffs.

Two limitations of these data are obvious. The sample on which the average prices are based may be biased; and the prices referred to are prices for the farmer, and clearly not the right prices for the feeding-stuffs manufacturer to use. The first objection is met to some extent by taking yearly averages, and hoping that some of the bias will come out in the wash; the second remains. However, no other prices seem available.

The monthly prices per ton were averaged for three twelve-monthly periods, and expressed in terms of pence per 100 lb., as shown in Table I.

TABLE I

Feeding-Stuff prices in three years.

Feeding-stuff	*Nov. 1950–Oct. 1951 pence per 100 lb.*	*Nov. 1951–Oct. 1952 pence per 100 lb.*	*Nov. 1952–Oct. 1953 pence per 100 lb.*
Oilcakes			
Linseed	382.7	409.0	432.8
Decorticated Groundnut ...	362.6	412.8	418.2
Undecorticated Groundnut ...	360.4	360.0[1]	359.0
Decorticated Cotton Cake ...	356.6	387.4	398.1
Undecorticated Cotton Cake ...	348.5	318.7	323.4
Coconut	348.4	391.7	382.8
Palm Kernel	334.7	378.2	363.8
Meals			
Flaked Maize	367.5	407.6	408.1
Maize Meal	354.4	397.4	398.2
Barley Meal	332.9	371.4	366.8
Fish Meal	482.9	557.1	578.1
Grains			
Wheat	290.0[1]	308.3	352.2
Barley	299.1	313.5	324.0
Oats	289.9	308.0	313.1
Crushed Oats	300.2	358.7	353.8
Maize	361.0	390.2	380.8
Milling Offals			
Bran	351.3	283.4	294.1
Weatings	258.0	281.7	294.1
Miscellaneous			
Dried Sugar Beet Pulp... ...	160.0	188.5	181.0
Dried Brewers' Grains	191.2	210.2	218.0

[1] Estimated, since data were not available.

The next step is to prepare a table showing the proportions of various dietary constituents contained in the twenty feeding-stuffs in Table I.[1] This information is given in Table II. The feeding stuffs contain definite quantities of other important dietary elements, such as calcium, chlorine, potash, etc., but these are ignored in our prescribed diets. This is partly for simplicity, but mainly because these elements are usually provided by various additives in the form of mineral salts.

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[1] These proportions were obtained from H. E. Woodman : *Rations for Livestock*, 11th Edition, Bulletin No. 48 of Ministry of Agriculture and Fisheries, H.M.S.O., 1948, pp. 107–127, and E. T. Halnan and F. H. Garner : *The Principles and Practice of Feeding Farm Animals* : Longmans, 1940, pp. 335–342. The former source was used whenever possible, since it is of more recent date.

Perhaps a note should be added concerning the meaning of such terms as ‘ starch equivalent ’. These are conventional measures used by experts in animal husbandry, and have well-defined meanings, which need not delay us here. It might be added that American usage is considerably different.

TABLE II

Dietary Constituents of Certain Feeding-Stuffs

Feeding Stuff	*Starch Equivalent* %	*Protein Equivalent* %	*Crude Fibre* %	*Phosphorus*[1] %
Oilcakes				
Linseed	74.0	24.6	9.1	1.7
Decorticated Groundnut ...	73.0	41.3	6.4	1.3
Undecorticated Groundnut ...	56.8	27.3	22.9	1.0
Decorticated Cotton Cake ...	41.6	17.3	21.3	2.5
Undecorticated Cotton Cake	68.4	34.3	7.8	2.7[2]
Coconut	76.8	16.4	11.5	1.5
Palm Kernel	73.2	17.0	13.4	1.1
Meals				
Flaked Maize[1]	84.0	9.2	1.5	0.6
Maize Meal	77.6	7.7	2.2	0.8
Barley Meal	71.4	7.3	4.5	0.8
Fish Meal	58.9	53.0	0	9.0
Grains				
Wheat	71.6	9.6	1.9	0.9
Barley	71.4	7.3	4.5	0.8
Oats	59.5	7.6	10.3	0.8
Crushed Oats	59.5	7.6	10.3	0.8
Maize	77.6	7.7	2.2	0.8
Milling Offals				
Bran	42.6	10.0	10.3	2.8
Weatings	56.5	10.9	6.0	2.6
Miscellaneous				
Dried Sugar Beet Pulp[3] ...	0	3.1	18.3	0.2
	60.6	5.2	18.3	0.2
Dried Brewers’ Grains ...	48.3	12.6	15.2	1.6

[1] Given in Halnan and Garner ; all other estimates from Woodman.
[2] Estimated. [3] For pigs the proportions in the upper row are to be taken.

Table II enables us to say that 100 lb. of, say barley, will contain 71.4 lb. of starch equivalent, 7.3 lb. of protein equivalent, 4.5 lb. of crude fibre, and 0.8 lbs. of phosphorus. The problem now remains of specifying the proportions of these various constituents that should be arrived at in a 100 lb. mix if the animal is to carry out its function satisfactorily.

There are many different diets for the same farm animal depending on the stage in the animal’s development, and on the work one asks it to do. After some consultation, I have chosen four fairly typical diets, which are derived from diets given in Woodman and Halnan and Garner, *op. cit.* The first two are for cows, the third for pigs and the fourth for poultry. These are arranged in ascending order of difficulty: the diet for cows has prescribed levels only for starch equivalent and protein equivalent, plus, of course, the weight limitation. The poultry diet, however, specifies levels for all four constituents and is therefore more difficult to compute.

The rations are as follows :

(*a*) This ration is for cows, and gives the weights of the various constituents required to produce one gallon of milk with a fat content of 3.5 —4 per cent. Halnan and Garner, p. 196, say that this requires 3.5 lb. of feed which should contain at least 2.5 lb. of starch equivalent and 0.6 lb. of protein. If this is converted into 100 lb. of feed we obtain the requirement that the 100 lb. sack should contain at least 71.4 lb. of starch equivalent, and at least 17.1 lb. of protein equivalent. The total mixture of feeding-stuffs should weigh exactly 100 lb.

(*b*) This diet is similar to the last, and is obtained from the same source, but gives a lower density for the constitutents. Instead of 2.5 lb. of starch equivalent and 0.6 lb. of protein equivalent in 3.5 lb. of feed, these need now only be found in 4 lb. of feed. Translated into 100 lb. sacks, these requirements become at least 51.3 lb. of starch equivalent and 15 lb. of protein equivalent. The basic feeding-stuffs can be drawn from all those in Table I, except fish meal, which has bad effects on the quality of the milk.

(*c*) The third ration, drawn from Woodman, p. 91, is for pigs at the final fattening weight of 200 lb. One hundred pounds of feed must contain at least 65 lb. of starch equivalent and 9 lbs. of protein equivalent, and not more than 6 lbs. of fibre content. (This last requirement is because fibre seems to be bad for pigs' stomachs.) The feeding stuffs available can be drawn from all those in Table I, except for linseed, undecorticated groundnut, cotton cake in any form, and coconut.

(*d*) The fourth diet is a maintenance and laying ration for a 5 lb. hen, the aim being that she should lay one egg every other day. This is taken from Woodman p. 106, and requires that in 100 lb. of feed there should be at least 58.8 lb. of starch equivalent and 7.3 lb. of protein equivalent, not more than 6.5 lb. of fibre content, and at least 0.7 lb. of phosphorus. The feeding stuffs available can be drawn from all those in Table I, with the same exceptions as for pigs.

PART IV. THE RESULTS

This section presents the solutions to the various least-cost diet problems posed in Part III, and gives the feeding-stuffs contained in each diet, their amounts, the total cost of each diet, and the amount of over-fulfilment of the basic constraints.

The diets will be taken in the original order, beginning with (*a*). This diet required a total weight of 100 lbs. exactly, a starch equivalent of at least 71.4 lb. and a protein equivalent of at least 17.1 lb.

At the prices prevailing in 1950–1, the cheapest diet consisted of 29.3 lb. of decorticated groundnut, 32.3 lb. of flaked maize and 38.3 lb. of dried sugar beet pulp. This diet which, allowing for round-off errors—a limitation of all this analysis—comes to 100 lb., cost 268 pence. It met the protein requirement of 17.1 lbs. exactly, and over-fulfilled the starch equivalent requirement by 0.3 lbs.

The diet was then recalculated at the 1951–2 prices, as a result of which decorticated groundnut left the diet, to be replaced by undecorticated cotton cake. This new ration consisted of 36.2 lb. of undecorticated cotton cake, 34.0 lb. of flaked maize and 29.8 lb. of dried sugar beet pulp. The cost of this mixture was 310.1 pence, and the dietary limitations were met exactly.

Recomputation with the 1952–3 prices made no change in the feeding-stuffs mixed in the ration, not did it change the proportions in which they entered the mixture. This illustrates the point mentioned in Part II, that changes in relative prices may not alter the optimum combination. The total cost fell a minute amount, from 310.1 pence per 100 lb. to 309.8 pence. Naturally, the dietary requirements continued to be met exactly.

Diet (*b*) was then computed. This, it will be remembered, specified a 100 lb. total weight, and lower levels of 51.3 lb. of starch equivalent and 15.0 lb. of protein equivalent. These limitations are less stringent than those for diet (*a*), and one would expect that the cheapest mixture for (*b*) would be considerably cheaper than that for (*a*), which was in fact the case.

In order to make life easier all round, the results of the remaining computations will be presented in more or less tabular form.

		Starch Equiv.	*Protein Equiv.*
1950-1 prices			
10.6 ℔ Decorticated Groundnut	containing	51.9 ℔	15.0 ℔
8.6 ℔ Dried Sugar Beet Pulp...			
80.6 ℔ Dried Brewers' Grains			
The total cost is 206.3 pence.			
1951-2 prices			
12.6 ℔ Undecorticated Cotton Cake	containing	51.3 ℔	15.0 ℔
4.5 ℔ Dried Sugar Beet Pulp			
82.7 ℔ Dried Brewers' Grains			
The total cost is 222.5 pence.			
1952-3 prices			
33.7 ℔ Undecorticated Cotton Cake	containing	63.2 ℔	15.0 ℔
66.4 ℔ Dried Sugar Beet Pulp			
The total cost is 229.2 pence.			

The diet calculated at the 1952–3 prices illustrates the case where the cheapest ration contains fewer feeding-stuffs than the number of constraints.

With the next diet (*c*), we move from cows to pigs, and increase the number of prior conditions by one. The 100 lb. weight limitation remains, and the lower limits for starch and for protein are 65.0 lb. and 9.0 lb. respectively. The new condition is a 6 lb. *upper* limit for fibre content.

The results, again in tabular form, are as follows :

		Starch Equiv.	*Protein Equiv.*	*Fibre Content*
1950-1 prices				
0.5 ℔ Decorticated Groundnut	containing	650.0 ℔	10.6 ℔	5.7 ℔
71.2 ℔ Wheat				
28.3 ℔ Dried Brewers' Grains				
The total cost is 262.4 pence.				
1051-2 prices				
As for 1950 prices, the total cost being 281.1 pence.				
1952-3 prices				
0.5 ℔ Decorticated Groundnut	containing	65.0 ℔	9.0 ℔	6.0 ℔
61.7 ℔ Barley				
27.2 ℔ Weatings				
10.6 ℔ Dried Brewers' Grains				
The total cost is 305.1 pence.				

These results provide additional illustrations of two phenomena that have already occurred : (i) the persistence of the same optimal ration after a change in relative prices, and (ii) the solution containing fewer feeding-stuffs than limitations. It is perhaps convenient to point out here that if *exact* fulfilment of the diet were always required, it would usually be more expensive than if a little overshooting of the mark were allowed of some constraints.

The last diet was that for laying hens, diet (*d*). This required a 100 lb. weight limitation, and lower levels of 58.8 lb. of starch equivalent and 7.3 lb. of protein equivalent. Not *more* than 6.5 lb. of fibre is allowed, and at least 0.7 lb. of phosphorus must be achieved. The most profitable rations are:

		Starch Equiv. ℔	*Protein Equiv.* ℔	*Fibre Content* ℔	*Phosphorus* ℔
1950-1 prices					
28.8 ℔ Wheat	containing	60.3	11.0	6.5	2.0
54.7 ℔ Weatings					
18.1 ℔ Dried Brewers' Grains					
The total cost is 259.2 pence.					
1951-2 prices					
As for 1950-1, the total cost now being 280.9 pence.					
1952-3 prices					
23.1 ℔ Barley	containing	59.8	10.3	6.5	1.1
68.9 ℔ Weatings					
9.0 ℔ Dried Brewers' Grains					
The total cost is 297.1 pence.					

There is one important point to notice about these results. The round-off errors are now quite considerable, being of the order of 1 per cent. This is to be expected, since it is intuitively clear that because we have a larger number of constraints, a larger number of iterations will usually be required, and hence a greater possibility of round-off error occurs. This turned out to be the case. All the computations were carried out using 3 places of decimals, but for larger problems more accuracy is certainly required. An error of 1 per cent is definitely too large for this problem, but, unfortunately, time was not available to recompute the diets using a more accurate number, say five or six, of decimal places. Since these examples are only illustrative, the loss is perhaps not very great; nevertheless greater accuracy should be sought in any practical problem.

I have avoided commenting in any detail on the set of results, although the reader may feel that they are already tiresome enough. The possible objections that may be levelled against them, which have been mentioned in Part III, still stand ; to them must be added the loss of accuracy due to round-off error.

One possible objection may be that it is not good policy, from both the manufacturing and the demand side, to change the physical constitution of one's compound from year to year. This objection clearly has some force, but the inconvenience of changing products must be balanced against the increase in cost because one fails to do so. In order to assess this increase in cost, I have computed Table III, which gives the cost in the other two years of the solution for any given year, for each of the four diets. Since some of the solutions persist for more than one year (there is none that remains unchanged throughout), some of the rows in Table III are duplicates.

Table III

	Cost in pence of 100 ℔ in		
	1950–1 prices	*1951–2 prices*	*1952–3 prices*
Diet (*a*) *Cows*			
1950–1 solution	268.2	334.8	333.7
1951–2 ,,	298.8	310.1	309.8
1952–3 ,,	298.8	310.1	309.8
Diet (*b*) *Cows*			
1950–1 solution	206.3	229.4	235.3
1951–2 ,,	209.2	222.5	229.2
1952–3 ,,	223.7	232.6	229.2
Diet (*c*) *Pigs*			
1950–1 solution	262.4	281.1	314.4
1951–2 ,,	262.4	281.1	314.4
1952–3 ,,	276.8	294.4	305.1
Diet (*d*) *Chickens*			
1950–1 solution	259.2	280.9	301.8
1951–2 ,,	259.2	280.9	301.8
1952–3 ,,	264.1	285.4	297.1

Table III shows that the additional cost incurred by not changing one's mixture varies from almost zero (the 1951–2 solution of diet (*b*) at 1952–3 prices differs only in the second decimal place from that of the 1952–3 solution) to around 10 per cent. Whether the increased cost justifies the change is something that can only be settled *ex post* i.e. after the new solution has been computed. The cost of the computation itself need not be very great, however.

It might be instructive to give the average prices for 100 lbs of various compounded feeding-stuffs, as given by the Ministry of Agriculture Feeding-Stuffs Prices Enquiry. These are, of course, not strictly comparable with the mixtures that this paper has computed, since they are the average prices of a large number of different compounds, each with its secret formula. The results might be of some interest, however, as indicating broad trends. The figures given for my ' home-made ' mixtures are for diets (*a*) (*c*) and (*d*), (diet (*b*) has been dropped since it is a ' diluted ' form of diet (*a*)).

TABLE IV. *Price Comparisons for certain mixtures*

Mixtures	*1950–1 prices*	*1951–2 prices* *pence per 100 lb*	*1952–3 prices*
Dairy Cakes and Meals ...	337.4	382.8	388.9
Computed Dairy Mix Diet (*a*) ...	268.2	310.1	309.8
Pig Meals	347.0	387.2	385.6
Computed Pig Mix Diet (*c*) ...	262.4	281.1	305.1
Poultry Meals	366.9	415.2	344.8
Computed Poultry Mix Diet (*d*)...	259.2	280.9	297.1

Although any detailed comparison between the two types of mixtures would certainly be absurd, I would venture the opinion that a mixing of the basic feeding-stuffs to form one's own compound meal would save something of the order of one quarter the cost of already compounded meals. However, this does not take into account the cost of computing or of the labour and trouble involved in mixing one's own feed; and these may well outweigh the saving in cost. In any case, this study is avowedly of more interest to the manufacturer than to the farmer.

PART V. CONCLUSIONS

The aim of this section is to mention briefly certain limitations of linear programming, and to indicate the lines of future research.

The first obvious limitation is that of linearity, both in the functional to be maximised, and in the inequalities by which the solution is constrained. Efforts have been made to generalise the problem by substituting non-linear functionals for the usual linear functionals. So far these treatments have yielded results only for very simple non-linear functionals[1] or have concerned themselves mainly with existence theorems.[2] Some recent unpublished work of E. M. L. Beale on convex functionals is of importance in this context. A much more difficult problem is that where the constraints are non-linear; I do not know of any work along these lines.

A further limitation has been mentioned already, and has to do with the problem of errors, both in the input coefficients, and in the specification of the inequalities. For example, in the empirical problems treated in the present paper, the diets presented have been drawn from textbooks on animal husbandry. How many grains of salt should be taken with these diets is not entirely clear, but it is clear that although based on careful experiments, they may be too rough and ready a guide really to justify very exact computations like those performed. Much remains to be done in investigating the effects of errors; some recent work by David Votaw (as yet unpublished except for abstracts)[3] is directed to this problem.

In spite of these drawbacks, however, linear programming represents a great advance in the ability of economics actually to give quantitative answers to important questions. Several interesting problems can already be tackled with simplex methods; future progress will uncover many more applications.

[1] R. Dorfman : *Application of Linear Programming to the Theory of the Firm* : University of California, 1951.

[2] H. W. Kuhn and A. W. Tucker : ' Non-linear Programming ' *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability* (ed. J. Neyman) : University of California, 1951, pp. 481–492.

[3] See D. F. Votaw : ' Statistical Programming ' : *Annals of Mathematical Statistics*, Vol. 25, No. 4, December 1954, p. 809.